Assesing the Economic Significance of the Intra-daily Volatility Seasonalities

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September 21, 2005

Abstract

It is a well established empirical fact that volatility follows approximately an inverted U-shaped pattern during the day. It is high in the morning, gradually decreasing, reaching a minimum at lunch time and then starting to increase again until the end of the trading day. In this paper we investigate the dynamic properties of these intra-daily volatility seasonalities. More specifically, we divide daily volatility into several parts and model them separately. Our analysis shows that morning/afternoon volatility has a different time-series behaviour in comparison to lunch time volatility. Also, a substantial improvement in forecasting performance can be obtained by partitioning daily volatility into parts which correspond to the observed intra-daily seasonalities.

1 Introduction

Trading in asset markets exhibits strong and persistent intra-daily seasonalities. More specifically, it is well-known that the volatility follows a U-shaped pattern during the day, i.e., it is high in the morning hours and then gradually decreases, reaching a minimum at lunch; after that, it starts to increase, reaching a maximum at the end of the trading day. This phenomenon was documented for the first time in the academic literature by Wood, McInish and Ord (1985). Jain and Joh (1988) give evidence that trading volume exhibits similar intra-daily seasonalities. Later, similar findings has been reported by many researchers, see, for example, Harris (1986), Admati and Pfleider (1988) and Foster and Viswanathan (1993).

Although the inverse U-shaped pattern of volatility and trading during the day is invariably present across different markets, the reasons for this phenomenon are not entirely understood. Adamti and Pfleider (1988) show that in a partial equilibrium setting, where uninformed liquidity traders have a discretion over the timing of their trades, periods of concentrated trading and volatility can arise, with heavy trading by both informed and uninformed traders. However, in their analysis, trade clustering appears only when the private signals of the informed traders are highly correlated. Also, the model of Adamti and Pfleider (1988) does not give predictions for the location of the time intervals with trading and volatility clustering.

In this paper, we consider intra-daily volatility seasonalities from a different angle. Instead of trying to explain theoretically the inverted U-shaped pattern, we investigate the economic significance of this phenomenon in terms of improving the forecasting performance of volatility models. The idea of our paper starts with the observation that daily volatility can be decomposed into several components, which correspond to the observed intra-daily regularities. For example, a natural choice for such decomposition would be to partition the daily volatility into morning, lunch and afternoon periods. The theory suggests (Adamti and Pfleider (1988)) that market participants may trade for different reasons in the time intervals corresponding to the different volatility components. Accordingly, the individual volatility components can exhibit distinctive time-series behavior and this can be exploited to obtain better models for the behavior of the daily volatility.

To confirm this hypothesis, we utilize several high-frequency volatility measures, which can be readily decomposed into intra-daily components. These intra-daily volatility components are then modelled in a multivariate econometric framework, which allows for lead-lag effects. As a next step, we evaluate the economic significance of this approach by comparing the volatility forecasts obtained from models which use as inputs only the daily volatility measures as opposed to volatility forecasts formed by summing the predictions for the individual components of these measures.

We consider several volatility measures calculated for the futures contract on the French stock market index CAC40 for the period between January 3, 2000 and December 31, 2004. The first is the realized volatility measure of Andersen, Bollerslev, Diebold and Labys (2001,2003) (henceforth ABDL), which is simply the sum of the intra-daily five minutes squared returns. To check for robustness of our results, we introduce another informal volatility measure called the realized range. It is defined as the sum of the extreme-value volatility measures in each of the five-minutes intra-daily intervals. For every five-minutes intra-daily interval, the extreme value volatility measure is the squared difference between the logarithm of the price maximum and the logarithm of the price minimum, multiplied by a constant (see Parkinson (1980)).

Our main modelling tool is the Multiplicative Error Model (MEM) of Engle (2002). It provides a general framework for modelling non-negative time-series and has been successfully used for volatility modelling and predictions (for example, Engle and Gallo (2003),Chou (2004) and Brunetti and Lildholdt (2002)). To measure the improvement or worsening in forecasting performance, we calculate the relative change in the Root Mean Squared Error (RMSE), using the models for the daily realized volatility and the daily realized range, respectively as benchmarks. Predictions for horizons of length between one and twenty days are calculated, since such time intervals are most relevant to practical risk-management issues. The results indicate a sharp decrease in the RMSE for long-term prediction horizons, reaching a reduction in the RMSE by a factor of almost a half for 20 days horizons.

We perform a similar analysis for the realized power measure, advocated by Ghysels and Forsberg (2004). It is defined as the sum of the absolute intra-daily returns. As shown by Ghysels and Forsberg (2004), this measure is much more persistent and predictable than the realized volatility and it is also immune to jumps (see Barndorff-Nielsen and Shephard(2004)). Although, not exactly a volatility measure, the realized power has been used successfully to predict the realized volatility in the paper by Ghysels, Santa-Clara and Valkanov (2004). We perform a similar analysis for realized power, like that for realized volatility and realized range, and find a significant improvement in performance, as in the first two cases.

Additionally, we take advantage of the recent theoretical results derived in a series of papers by Barndorff-Nielsen and Shephard(2002,2004), who introduced a series of high-frequency volatility and volatility related measures, with a rigorous asymptotic theory accompanying them. More specifically, working in a quite general framework and allowing for discontinuous jumps in prices, they show that realized volatility measure can be decomposed into a sum of two parts. The first one, which they call the realized bi-power accounts for the volatility, due to the continuous changes in the price process. The second one, which we refer to as the realized jump, is the difference between the realized volatility and the realized bi-power, and accounts for the volatility due to jumps in the price process.

The former two measures can be readily decomposed into intra-daily components. Again, we compare the forecasts for the daily and component MEM models and find significant improvement in performance for longer forecasting horizons for both measures. Note that, from a practical point of view, we are interested in modelling and predicting realized volatility, since its conditional expectation is approximately equal to the conditional return variance. Since the realized volatility is the sum of the realized bi-power and realized jump, we can try to improve the forecasts for realized volatility by modelling the realized bi-power and realized jump separately, while accounting for possible lead-lag relationships between the two measures. This idea has been implemented by Andersen, Bollerslev and Diebold (2003). They find that separating the realized volatility into realized bi-power and realized jump in a simple autoregressive model for the realized volatility leads to better forecasts.

To compare how the approach of Andersen, Bollerslev and Diebold (2003) compares to our approach, we obtain predictions for the realized volatility by summing the forecasts from the models for the realized bi-power and realized jump. This leads to an improvement in performance similar to that of the model for the intra-daily components of the realized volatility. Finally, to see whether combining our approach and the approach of Andersen, Bollerslev and Diebold (2003) results in further economic gains, we sum the predictions of the models for the intra-daily components of the realized bi-power and realized jump. However, in that case the forecasting performance not only does not improve but it actually deteriorates. A possible reason could be the large number of parameters of the combined model.

The rest of the paper is organized as follows. In Section 2 we give an overview of the different volatility measures which we use in this study. Section 3 contains a description of the data set and some preliminary statistics. The Multiplicative Error Model (MEM) of Engle (2002) is discussed in Section 4. In-sample estimations of the MEM specifications for the different volatility measures are presented in Section 5. Comparison of the forecasting performance across the competing volatility models is given in Section 6. Finally, a conclusion and some directions for future research are given in Section 7.

2 Realized Volatility, Realized Bi-Power, Realized Jump, Realized Power and Daily Range as Proxies for the Latent Market Volatility

In this section, the notion of realized volatility is introduced, which is an unbiased measure for the unobserved market volatility, under some loose conditions and has wide applications for volatility modelling. Next, two new volatility measures, realized bi-power and realized jump, derived recently by Barndorff-Nielsen and Shephard (2004), are discussed. These two measures allow one to isolate the impact of discrete jumps in prices on volatility in a non-parametric way. As an additional robustness check of our results, the power variation measure of Barndorff-Nielsen and Shephard (2004) is presented. Finally, we define an *ad-hoc* volatility measure called the realized range. All of these five measures are calculated from high-frequency data and can be readily divided into intra-daily components.

The literature on using high-frequency data for volatility modelling virtually started with the papers by ABDL(2001,2003) where these authors formally introduced the notion of realized volatility. As we mentioned before, realized volatility is simply the sum of the squared intra-daily returns sampled at some small fixed interval such as five minutes. Although the definition is simple, the theory related to the use of realized volatility as a proxy for the unobservable market volatility is more complicated, with some recent breakthroughs, on which we comment below.

To fix ideas, let $\{p_t\}, t \in [0, \infty)$, be the logarithm of the price process $p_t = \log(P_t)$ of a given asset. We denote the information set at time t by $\mathcal{F}_t$. The daily return on day m, where $m \geq 1$ is an integer, is defined as $r_m = p_m - p_{m-1}$. Central to the notion of volatility is the quadratic variation $[r_t^*, r_t^*]$ of the cumulative return process $r_t^* = p(t) - p(0)$, given by the expression:

$$[r_t^*, r_t^*] = (r_t^*)^2 - 2 \cdot \int_0^t r_s^* \partial r_s^*. \quad (1)$$

The intra-daily returns sampled at some fixed time interval with length $0 < \Delta < 1$ are denoted by $r_{m,j} = p_{m+j*\Delta} - p_{m+(j-1)*\Delta}$ for $j = 1, ..., 1/\Delta$ (without loss of generality we assume that $1/\Delta$ is an integer for the ease of notation). Then realized volatility on day $m+1$ sampled at the $1/\Delta$ frequency is defined as:

$$RV_{m+1} = \sum_{j=1}^{1/\Delta} r_{m,j}^2. \quad (2)$$

By the theory of quadratic variation, realized volatility RV_{m+1} converges in probability to the increment in the quadratic variation of the return process r_t^*

$$RV_{m+1} \rightarrow \left[r_{m+1}^*, r_{m+1}^*\right] - \left[r_m^*, r_m^*\right], \quad (3)$$

when $\Delta \rightarrow 0$ under some very loose conditions (see ABDL (2001,2003)). In a recent paper, Barndorff-Nielsen and Shephard (2002) derive the asymptotic distribution of the realized volatility and its rate of convergence to the quadratic variation, working in a stochastic volatility framework. On the other hand, as discussed in ABDL (2001,2003), the conditional value of market volatility coincides with the conditional value of the increment in the quadratic variation for a wide class of return processes, including those with jumps and non-Markov transition probabilities. For example, the conditional variance of the cumulative return between the days $m+1$ to $m+p$ on day m equals the conditional value of the increment in the quadratic variation

$$Var\left(r_{m+1} + ... + r_{m+p} | \mathcal{F}_m\right) = E\left(\left[r_{m+p}^*, r_{m+p}^*\right] - \left[r_m^*, r_m^*\right] | \mathcal{F}_m\right). \quad (4)$$

On the other hand, $\left[r_{m+p}^*, r_{m+p}^*\right] - \left[r_m^*, r_m^*\right] \approx RV_{m+1} + ... + RV_{m+p}$. We can re-write expression (4) as

$$Var\left(r_{m+1} + ... + r_{m+p} | \mathcal{F}_m\right) \approx E\left(RV_{m+1} + ... + RV_{m+p} | \mathcal{F}_m\right). \quad (5)$$

Instead of modelling the market volatility, we can model realized volatility, and expression (5) guarantees that their conditional values are approximately equal. Realized volatility has the advantage that it is an observable variable and can be readily computed from high-frequency data.

As the size of the sampling interval goes to zero, realized volatility converges to the quadratic variation for a wide class of processes, which includes those with jumps. For such types of processes, the quadratic variation as well as the realized volatility, consists of a "pure stochastic volatility" part and a "jump" part. To illustrate this point, consider the following general specification for the return process, which is frequently employed in the continuous-time finance:

$$\partial r_t^* = \mu(t)\,\partial t + \sigma(t)\,\partial W(t) + J(t)\,\partial q(t)\,, \tag{6}$$

where $\mu(t)$ and $\sigma(t)$ are continuos functions. Here $W(t)$ is a Brownian motion, and $q(t)$ is a counting process, such that $\partial q(t) = 1$ if there is a jump at time t and $\partial q(t) = 0$ otherwise. The jump intensity is given by a time-varying function $\lambda(t)$ and the jump sizes $J(t)$ are stochastic. Let $0 < t_1 < t_2 < ....$ be the times when jumps occur, and let $j(t_1), j(t_2), ...$ be the corresponding jump sizes $j(t_i) = r_{t_i}^* - r_{-t_i}^*$. From the general theory of stochastic processes it follows

$$[r_t^*, r_t^*] = \int_0^t \sigma^2(s)\,\partial s + \sum_{t_i \le t} j^2(t_i)\,. \tag{7}$$

Since

$$RV_{m+1} \approx \left[r_{m+1}^*, r_{m+1}^*\right] - [r_m^*, r_m^*] = \int_m^{m+1} \sigma^2(s)\,\partial s + \sum_{m \le t_i \le m+1} j^2(t_i)\,, \tag{8}$$

it is clear that realized volatility incorporates both the part of the quadratic variation given by $\int_m^{m+1} \sigma^2(s)\,\partial s$, which corresponds to the continuous part of the return generating process, as well as the part of the quadratic variation $\left(\sum_{m \le t_i \le m+1} j^2(t_i)\right)$, which is caused by the presence of jumps in the return generating process. Barndorff-Nielsen and Shepherd (2004) propose a non-parametric way to disentangle these two components of realized volatility. They introduce a new measure, called the realized bi-power defined as

$$RBP_{m+1} = \frac{1}{\sqrt{2\pi}} \sum_{j=2}^{1/\Delta} |r_{m,j}| \cdot |r_{m,j-1}|\,. \tag{9}$$

The authors show that (again) under some loose conditions, the realized bi-power converges to the integrated volatility $\int_0^t \sigma^2(s)\,\partial s$ when the return process is of the type (8). The difference between the realized volatility and realized bi-power converges in probability to the jump component of the quadratic volatility

$$RV_{m+1} - RBP_{m+1} = RJ_{m+1} \rightarrow \sum_{m \le t_i \le m+1} j^2(t_i)\,. \tag{10}$$

We call the variable RJ_{m+1} the realized jump. Note that in (10), nothing prevents the realized jump measure from becoming negative. Following the suggestions of Barndorff-Nielsen and Shephard (2004), we set RJ_{m+1} to zero when the difference $RV_{m+1} - RBP_{m+1}$ is negative (note that, in our data set, such instances are very rare).

Besides its theoretical significance, this decomposition could have important applications for modelling and forecasting. Andersen, Bollerslev and Diebold (2003) show that including the jump component in a simple reduced model for realized volatility results in a significant improvement in the forecasting performance. In this paper we will further elaborate this approach by additionally decomposing the realized bi-power and jump into intra-daily components and modeling them separately, while accounting for possible lead-lag effects. Additionally, we use the MEM framework of Engle (2002) to model the joint behavior of these two volatility measures instead the simple linear regression approach of Andersen, Bollerslev and Diebold (2003).

To check the robustness of our results, we also utilize another volatility measure introduced by Barndorff-Nielsen and Shephard (2004) called the power variation measure, which is, like the realized bi-power, immune to jumps in if the return process is of the type given in equation (6). More specifically, the power variation is defined in exactly the same way as the realized volatility, with the only difference being that we sum over the intra-daily absolute returns, not over the squared intra-daily returns:

$$RP_{m+1} = \sum_{j=1}^{1/\Delta} |r_{m,j}| \, . \tag{11}$$

The authors showed when the return generating process is of the type given in equation (6), then

$$RP_{m+1} \to \int\limits_{m}^{m+1} \sigma\left(s\right) \partial s, \tag{12}$$

when $\Delta \to 0$ under some very loose conditions. Obviously, the power variation is insensitive to the presence of jump component, although it does not converge exactly to the integrated volatility $\int\limits_{m}^{m+1} \sigma^2\left(s\right) \partial s$.

Nevertheless, in a recent study by Ghysels, Santa-Clara and Valkanov (2004), it is shown that realized power is an extremely good predictor of future realized volatility when used as a regressor in a Mixed Data Sampling (MIDAS) regression framework. Similar findings are reported by Forsberg and Ghysels (2004). This is a bit surprising, since realized power and realized volatility have different scales. One possible (and incomplete) explanation is that realized power, besides being immune to jumps, is much more persistent and has better sampling error properties than the other volatility proxies such as realized volatility and realized bi-power (see Forsberg and Ghysels (2004)). For these reasons, we include realized power in our analysis and consider whether it is possible to improve the forecasting performance by dividing this measure into intra-daily components.

As a last robustness check, we introduce an ad-hoc high-frequency volatility proxy based on the daily-range measure. To fix ides, let the log-price p_t follows a driftless Arithmetic Brownian Motion process with a constant volatility in the interval $[\delta_1, \delta_2]$ given by

$$\partial p_t = \sigma_{[\delta_1,\delta_2]} \partial W_t,\ t \in [\delta_1, \delta_2]\,. \tag{13}$$

One obvious estimator for the volatility parameter $\sigma^2_{[\delta_1,\delta_2]}$ is given by $\frac{1}{\delta_2-\delta_1} \cdot (p_{\delta_2} - p_{\delta_1})^2$. Parkinson (1980) introduced an alternative estimator based on the price range given

$$R_{[\delta_1,\delta_2]} = \sup_{\delta_1 \leq t \leq \delta_2} p_t - \inf_{\delta_1 \leq t \leq \delta_2} p_t \tag{14}$$

of the volatility defined as

$$\widetilde{\sigma}^2_{[\delta_1,\delta_2]} = \frac{1}{4\ln(2)} \cdot R^2_{[\delta_1,\delta_2]}. \tag{15}$$

He showed that it is an unbiased estimator of $\sigma_{[\delta_1,\delta_2]}$, which is about five times more efficient than the naive $\frac{1}{\delta_2-\delta_1} \cdot (p_{\delta_2} - p_{\delta_1})^2$ estimator. There is evidence that the estimator given in equation (15) preserves its desirable properties even if the volatility in the interval $[\delta_1, \delta_2]$ is stochastic (see Alizadeh, Brandt and Diebold (2002)). Since this measure is usually applied to daily data, it is customary called the daily range.

We adopt the approach of using the range based volatility measure given in equation (15) in a high-frequency setting as follows. Suppose that the intra-daily returns are uncorrelated with each other; then the volatility of the daily returns is simply the sum of the volatilities of the intra-daily returns. One the other hand, the volatility in each of the intra-daily intervals can be derived from the range based estimate given in equation (15). This gives us a new high-frequency volatility measure, which we call the realized range, defined as

$$RR_{m+1} = \frac{1}{4\ln(2)} \cdot \sum_{j=1}^{1/\Delta} RR^2_{m,j}, \tag{16}$$

where

$$RR_{m,j} = \sup_{m+(j-1)*\Delta \leq t \leq m+j*\Delta} p_t - \inf_{m+(j-1)*\Delta \leq t \leq m+j*\Delta} p_t. \tag{17}$$

In summary, in this section we discussed five high-frequency volatility measures: realized volatility, realized bi-power/ realized jump, realized power and realized range. All of these measures can be divided into intra-daily components, which we will later model separately. In order to get an intuition how to choose a partition of these daily volatility measures into intra-daily components, in the next section we take a closer look at the behavior of the individual components of these measures in a particular market.

3 Data Description and Preliminary Statistics for Intra-daily Volatility Seasonalities

The data used in this paper consist of all transactions on the futures written on the main French stock market index CAC40 and traded on the electronic trading platform MATIF. It spans the interval from January 3, 2000 to December 30, 2004, which consists of 1299 trading days. We consider only the nearest-to-maturity futures contract, which is invariably the most liquid one. The only exception happens on the maturity day of the contract, when it is traded until 11:00 AM. In that case, we switch to the next to maturity contract. Additionally, we discard 90 days from the sample (such as Christmas and New Year's Eve days) on which there is not enough trading, in order to calculate reliable estimate for the considered volatility measures. This leaves us with a total of 1209 observations.

The official trading day starts at 9:00 AM and ends at 5:30 PM. However, there is a pre-trading period with a starting time which varies between 7:00 AM and 8:00 AM and the trading can end as late as 10:00 PM. In accord with other studies on high-frequency financial data, we give attention only to the official opening hours, in which almost all of the trading is concentrated.

To gauge the nature of the intra-daily volatility patterns, we partition the interval from 9:00 AM to 5:30 PM into 102 five-minutes intervals, and compute the futures returns in each of these intervals for each trading day. Then we calculate the mean value of the squared returns for every five-minute interval across all 1209 trading days. The results are shown in Graph 1.

As it can be seen, the volatility during the day exhibits a strong seasonality, having a distinct behavior during each of the morning, lunch time and afternoon periods. We mentioned before that authors, who have previously dealt with this effect, usually describe it as an inverted U-shaped volatility pattern. However, from the graph above, it is clear that the actual pattern is more complex than an inverted U. In particular, the volatility gradually decreases during the morning hours, reaching its minimum at about 12:30. This pattern of decrease is rather smooth and roughly corresponds to the first part of an inverted U letter. During lunch time, that is between 12:30 and 14:30 PM, the volatility is at its lowest average levels for the day. Its mean value is relatively constant between 12:30 and 1:30 PM , then makes a slight upward jump and stays again relatively constant between 1:30 PM and 2:30 PM.

The afternoon period shows more erratic volatility behavior. We can divide this time interval roughly into three parts. The first one spans the period between 2:30 PM and 3:30 PM when the mean value of the volatility makes a jump upward and slightly decreases till 3:30 PM. The second one spans the time between 3:30 PM and 4:30 PM when the average volatility increases again and has a relatively unstable behavior till the end of the period at 4:30 PM. The last period is between 4:30 PM and 17:30 PM, when the mean volatility reaches a roughly constant level, lower than that during the previous period.

An interesting feature of the data is the two very large peaks in the five-minutes intervals [2:30 PM; 2:35 PM] and [4:00 PM; 4:05 PM]. Since these are "round" hours, one can hypothesize that such sharp increases are due to news releases. However, most of the news releases are made in the morning, and we do not observe any such volatility peaks then[1]. Also, the sharp difference between the behavior of the average volatility in the morning and in the afternoon is another interesting phenomenon, for which we do not have an explanation.

In a nutshell, we can divide the trading day into six sub-periods such that, in each one of them, the average realized volatility has a distinctive behavior. These intra-daily volatility patterns are qualitatively stable across sub-samples. Graph 2 and Graph 3 show the same descriptive intra-daily statistics, which correspond to the realized range and the realized bi-power. They appear similar to the statistics for the realized volatility. Again, we can isolate six sub-periods, where the corresponding intra-daily volatility components exhibit a distinctive behavior.

Graph 4 presents similar statistics for the realized power. Note that in this case, the intra-daily behavior of the individual components (absolute five minutes returns) of this measure is slightly different. Namely, there seems to be not much difference between the mean values of the realized power during the two lunch intervals [12:30 AM ; 1:30 PM] and [1:30 AM ; 2:30 PM], and the two afternoon intervals [3:30 AM ; 4:30 PM] and [4:30 AM ; 5:30 PM]. Also, the Graph 4 is generally "smoother" than the Graphs 1-3. One possible explanation for this is the observation by Forsberg and Ghysels (2004) that absolute returns and realized power are more insensitive to jumps in the price process than the other volatility proxies which we consider.

As a final check for the robustness of the intra-daily volatility patterns, we consider the intra-daily seasonality of the trading volume. It is well-known that a strong positive relation between volume and volatility exists; see for example Lamoureux and Lastrapes (1990). Also, as documented by Jones, Kaul and Lipson (1994), the volatility-volume relation virtually disappears once a control for the transactions volume (the number of transactions) is included. More specifically, the latter authors show that the size of the transactions contains no additional information about the volatility beyond that contained in the number of transactions.

Guided by these findings, we compute the average number of transactions in each of the 102 intra-daily five-minutes intervals. The results are shown in Graph 5. As it can be seen, the intra-daily pattern of transactions volume is qualitatively very similar to the intra-daily patterns of the considered volatility measures, namely the realized volatility, the realized range and the realized bi-power.

The preliminary statistics above suggest that the volatility has a distinctive behavior during six consecutive intervals of the day. Now, we give some simple statistics for the volatility behavior in these intra-daily subperiods. For each day $t = 1, ..., 1209$ in the sample, let us denote the six intra-daily components of the realized volatility RV_t on that day by RV_t^m, RV_t^l, RV_t^{l1}, RV_t^a ,RV_t^{a1} and

[1] The author is gratefull to Stan Hurn for making this point.

RV_t^e. Here RV_t^m is the morning realized volatility and corresponds to the time interval [9:00 AM; 12:30]. The volatility during lunch time is captured by the terms RV_t^l and RV_t^{l1}, where RV_t^l corresponds to the time interval [12:30; 1:30 PM] and RV_t^{l1} corresponds to the time interval [1:30 AM ; 2:30 PM]. Afternoon volatility is captured by the terms RV_t^a and RV_t^{a1}, where RV_t^a corresponds to the time interval [2:30 AM ; 3:30 PM] and RV_t^{a1} corresponds to the time interval [3:30 AM ; 4:30 PM]. The evening volatility in the time interval [4:30 AM ; 5:30 PM] is captured by the term RV_t^e.

We use the same superscripts to denote the intra-daily components of the other volatility measures. For example, the intra-daily components of the realized bi-power RBP_t are denoted by RBP_t^m, RBP_t^l, RBP_t^{l1}, RBP_t^a, RBP_t^{a1} and RBP_t^e. It is useful to introduce notation for the transactions volume (the number of transactions). For every day $t = 1, ..., 1209$ in the sample, we denote the number of transactions on that day in the time interval [9:00 AM; 5:30 PM] by TR_t and the corresponding intra-daily components by TR_t^m, TR_t^l, TR^{l1}, TR_t^a, TR_t^{a1} and TR_t^e.

Since the morning sub-period is 3.5 hours long, and the other sub-periods are 1 hour long, the intra-daily volatility and transaction volume, which account for the morning time interval, are scaled by a factor of $\frac{1}{3.5}$, i.e., RV_t^m is transformed to $\frac{1}{3.5} \cdot RV_t^m$ and so on. In Table 1 we report the mean values, min-max range, median and the standard deviation of the considered variables. Interestingly, the standard deviation of the intra-daily components of the volatility measures follows similar pattern as their mean values, e.g. it is high in the morning, decreases during lunch time and increases in the afternoon. This suggests that the fourth moment of the return distribution, which is related to the "volatility of volatility" may also exhibit a robust intra-daily seasonality.

Statistics for the correlations between the volatility components are presented in Table 2. There seems to be no discernible, robust patterns besides that the morning volatility is highly correlated with the early lunch ([12:30 ; 1:30 PM]) and late afternoon/evening volatility ([3:30 AM ; 5:30 PM]) and less correlated with the late lunch/early afternoon volatility ([1:30 AM ; 3:30 PM]). This is despite, that the late afternoon/evening time interval is more distant from the morning period than the late lunch/early afternoon time interval. The pattern holds across all volatility measures. One explanation for this commonality in volatility during the beginning and the end of the trading day could be due to the differences in the characteristics of the market participants who trade during this period, and those who trade around midday (see Admati and Pfleiderer (1988)). In Section 5, we document similar behavior of the lead-lag relationships between the volatility components.

4 Multiplicative Error Model (MEM)

In this section we describe how to estimate a multivariate econometric model and obtain forecasts for the individual volatility components. These forecasts can be summed in order to obtain predictions for the daily volatility measures.

Our main tool is the Multiplicative Error Model (MEM), proposed by Engle (2002) as a general approach to modelling time series with non-negative elements. It is basically a variant of the classical GARCH model and identical to the Autoregressive Conditional Duration (ACD) model of Engle and Russel (1998). The MEM model has been successfully applied to describe the dynamics of the realized volatility as well as the dynamics of the daily squared returns and the daily range in the paper by Engle and Gallo (2003). Chou (2001) and Brunetti and Lildholdt (2002) consider modelling the daily range in the same framework. These findings indicate the potential usefulness of the MEM model as a tool to model the conditional volatility.

To fix ideas, let $RV_t, t = 1, .., N$ ($N = 1209$) be the realized volatility time-series and $\mathcal{F}_t$ be the information set at time t. In the MEM framework, the dynamics of the realized volatility is given by

$$RV_t = \mu_t \cdot \varepsilon_t, t = 1, ..., N, \text{ and } \mu_t = E[RV_t|\mathcal{F}_{t-1}]. \tag{18}$$

The error terms ε_t are non-negative i.i.d. random variables such that

$$E[\varepsilon_t|\mathcal{F}_{t-1}] = E_{t-1}[\varepsilon_t] = 1. \tag{19}$$

As in the GARCH and ACD cases, the conditional mean μ_t is set to follow an autoregressive process given by

$$\mu_t = \alpha + \beta \cdot \mu_{t-1} + \gamma \cdot RV_{t-1}. \tag{20}$$

Generally, we can put more lagged values of μ_t and RV_t in (20). However, experience shows that, as in the case of the daily GARCH model, the inclusion of additional lagged terms does not make much difference. From a statistical point of view, the coefficients for the additional lagged regressors are insignificant, and from economic point of view, there is a little difference in model's performance both in- and out-of-sample.

Intuitively, the conditional mean μ_t captures the path-dependence in the realized volatility since the scaled realized volatilities $\frac{RV_t}{\mu_t}$ should be independent if the model is properly specified. Indeed, a good diagnostic check of the MEM model is test whether the scaled realized volatilities $\frac{RV_t}{\mu_t}$ are serially correlated.

If we know the distribution of the residuals $\varepsilon_t, t = 1, ..., N$, we can easily derive the likelihood function and estimate the model via maximum likelihood. In practice, however, we do not have any theoretical guidance, how to choose the density function of the error terms. Engle and Gallo (2003) show that if the disturbance terms are Gamma distributed with mean one, then the parameter which determines the shape of the error term distribution does not affect the estimates for the coefficients α, β and γ in Newton-type procedures for maximizing the likelihood function.

Additionally, the authors propose to estimate the model (18-20) as a GARCH model by putting $\sqrt{RV_t}$ in place of the daily return, i.e., specifying the dynamics of $\sqrt{RV_t}$ as:

$$\sqrt{RV_t} = \sqrt{\mu_t} \cdot \varepsilon_t, \; t = 1, ..., N \;\text{ and } \varepsilon_t \in N\,[0, 1] \tag{21}$$

$$\left(\sqrt{\mu_t}\right)^2 = \alpha + \beta \cdot \left(\sqrt{\mu_{t-1}}\right)^2 + \gamma \cdot \left(\sqrt{RV_{t-1}}\right)^2. \tag{22}$$

From a statistical point of view, this specification is incorrect, since the values of $\sqrt{RV_t}$ are positive. However, estimating the "wrong" model (21-22) by Maximum Likelihood with a Newton-type of optimization procedure, using the existing GARCH software for example, results in the same parameter estimates for α, β and γ when we estimate the "correct" model (18-20) with Gamma-distributed error terms. Moreover, estimated parameter variance-covariance matrices will also coincide if we use the Bollerslev and Wooldridge (1992) robust variance-covariance matrice. Engle and Gallo (2003) further argue that estimating the "wrong" model (21-22) in that way gives consistent estimates for the parameters of the "correct" model (18 -20), even if the disturbance terms are not Gamma distributed. For more information, see Engle (2002) and Engle and Gallo (2003).

In the MEM framework, we can easily model the components of the daily realized volatility, while accounting for possible lead-lag interactions between them. We introduce another notation by denoting the intra-daily realized volatility components RV_t^m, RV_t^l, RV_t^{l1}, RV_t^a ,RV_t^{a1}, RV_t^e by RV_t^1, RV_t^2, RV_t^3, RV_t^4 ,RV_t^5, RV_t^6. In our "dissaggregated" multivariate MEM framework, the dynamics of the each component of the realized volatility is given by the following specification:

$$RV_t^i = \mu_t^i \cdot \varepsilon_t^i, i = 1, ..., 6; t = 1, ..., N \tag{23}$$

where

$$\mu_t^i = E[RV_t^i | \mathcal{F}_{t-1}] \text{ for } i = 1, ..., 6; t = 1, ..., N. \tag{24}$$

Each of the six residuals series $\left\{\varepsilon_{t,t=1,...,N}^i\right\}, i = 1, ..., 6$, consists of identically and independently distributed random variables such that $E[\varepsilon_t^i | \mathcal{F}_{t-1}] = 1$ for $i = 1, ..., 6$, $t = 1, ..., N$. An autoregressive specification for the conditional mean values is specified, which allows for lead-lag effects:

$$\mu_t^i = \alpha_i + \beta_i \cdot \mu_{t-1}^i + \sum_{j=1}^{k} \gamma_{i,j} \cdot RV_{t-1}^j, \text{ for } i = 1, ..., 6 \text{ , } t = 1, ..., N. \tag{25}$$

Using (23) and (25), the conditional densities of each individual volatility component $f\left(RV_t^j | \mathcal{F}_{t-1}\right)$, $t = 1, ..., N$, can be derived in a straightforward way, as in the univariate case. This gives us a way to estimate the model (23-25) by separately estimating models for each volatility component via (quasi)maximum likelihood.

Potentially, a more efficient way to proceed with estimation would be to use the conditional joint density function of all six volatility components $f\left(RV_t^1, ..., RV_t^6 | \mathcal{F}_{t-1}\right)$, $t = 1, ..., N$. However, such an approach requires the specification of the joint density function of the residuals $\varepsilon_t^1, ..., \varepsilon_t^6$ for each $t = 1, ..., N$. Since we do not have any guidance over how to choose the correlation structure between the disturbance terms, we prefer to leave their joint density function unspecified. Our approach to estimating the multivariate MEM model for the realized volatility components follows that of Engle and Gallo (2003).

We derive the same multivariate models for the components of the other volatility measures. The multi-step forecasts in the MEM framework can be derived in a straightforward manner, as in the GARCH and ACD cases (see, for example, Engle and Russel (1998)). Since the transactions volume also exhibits intra-daily seasonalities, one could try to apply the same idea to its intra-daily components. However, it seems that the MEM model is highly mis-specified for modelling the transactions volume, since the fitted residuals show a high degree of autocorrelation. Perhaps a MEM model with long-memory similar to that of Brunetti and Lildholdt (2002) would be more appropriate. This is best left for future research.

5 In-Sample Estimation of the Component MEM model for the Volatility Measures

To gauge the dynamic relationships between the components of the volatility measures defined in Section 2, for each such measure we estimate six separate MEM models, one for each intra-daily component. First we consider the realized volatility and the realized range. For example, for the realized volatility we estimate the following MEM specifications:

$$E[RV_t^m|\mathcal{F}_{t-1}] = \alpha_1+\beta_1\cdot E[RV_{t-1}^m|\mathcal{F}_{t-2}]+\gamma_{1,1}\cdot RV_{t-1}^m+\gamma_{1,2}\cdot RV_{t-1}^l+\gamma_{1,3}\cdot RV_{t-1}^{l1} +\gamma_{1,4}\cdot RV_{t-1}^a+\gamma_{1,5}\cdot RV_{t-1}^{a1}+\gamma_{1,6}\cdot RV_{t-1}^e \tag{26}$$

$$E[RV_t^l|\mathcal{F}_{t-1}] = \alpha_2+\beta_2\cdot E[RV_{t-1}^l|\mathcal{F}_{t-2}]+\gamma_{2,1}\cdot RV_{t-1}^m+\gamma_{2,2}\cdot RV_{t-1}^l+\gamma_{2,3}\cdot RV_{t-1}^{l1} +\gamma_{2,4}\cdot RV_{t-1}^a+\gamma_{2,5}\cdot RV_{t-1}^{a1}+\gamma_{2,6}\cdot RV_{t-1}^e \tag{27}$$

$$E[RV_t^{l1}|\mathcal{F}_{t-1}] = \alpha_3+\beta_3\cdot E[RV_{t-1}^{l1}|\mathcal{F}_{t-2}]+\gamma_{3,1}\cdot RV_{t-1}^m+\gamma_{3,2}\cdot RV_{t-1}^l+\gamma_{3,3}\cdot RV_{t-1}^{l1} +\gamma_{3,4}\cdot RV_{t-1}^a+\gamma_{3,5}\cdot RV_{t-1}^{a1}+\gamma_{3,6}\cdot RV_{t-1}^e \tag{28}$$

$$E[RV_t^a|\mathcal{F}_{t-1}] = \alpha_4+\beta_4\cdot E[RV_{t-1}^a|\mathcal{F}_{t-2}]+\gamma_{4,1}\cdot RV_{t-1}^m+\gamma_{4,2}\cdot RV_{t-1}^l+\gamma_{4,3}\cdot RV_{t-1}^{l1} +\gamma_{4,4}\cdot RV_{t-1}^a+\gamma_{4,5}\cdot RV_{t-1}^{a1}+\gamma_{4,6}\cdot RV_{t-1}^e \tag{29}$$

$$E[RV_t^{a1}|\mathcal{F}_{t-1}] = \alpha_5+\beta_5\cdot E[RV_{t-1}^{a1}|\mathcal{F}_{t-2}]+\gamma_{5,1}\cdot RV_{t-1}^m+\gamma_{5,2}\cdot RV_{t-1}^l+\gamma_{5,3}\cdot RV_{t-1}^{l1} +\gamma_{5,4}\cdot RV_{t-1}^a+\gamma_{5,5}\cdot RV_{t-1}^{a1}+\gamma_{5,6}\cdot RV_{t-1}^e \tag{30}$$

$$E[RV_t^e|\mathcal{F}_{t-1}] = \alpha_6+\beta_6\cdot E[RV_{t-1}^e|\mathcal{F}_{t-2}]+\gamma_{6,1}\cdot RV_{t-1}^m+\gamma_{6,2}\cdot RV_{t-1}^l+\gamma_{6,3}\cdot RV_{t-1}^{l1} +\gamma_{6,4}\cdot RV_{t-1}^a+\gamma_{6,5}\cdot RV_{t-1}^{a1}+\gamma_{6,6}\cdot RV_{t-1}^e \tag{31}$$

In principle, one can include more lagged values in the equations (26-31), but as in the GARCH case these new terms are statistically insignificant and impact little the values of the forecasts. In Table 3 and Table 4 we present estimation of the multivariate MEM models for the components of the realized volatility and realized range, respectively. We estimate these models in way that is described in Section 4, using the Bollerslev and Wooldridge (1992) robust standard errors. As a specification test, the p-values for the Ljung-Box statistics, up to a lag 10 applied to the fitted residuals, are also included. For all models, we cannot reject the hypothesis that the fitted residuals are uncorrelated up to lag 10 and even at higher lags (these statistics are not reported). Only the model for the late lunch/early afternoon realized range shows a slight misspecification, since the corresponding p-values for the Ljung-Box statistics are greater than the 5% critical value. However, the autocorrelations at different lags do not seem to be very high.

To get a better picture for the relations between the volatility components, we a perform a Wald test for the joint significance of coefficients at a 5% percent significance level, and report the restricted models. As it can be seen, there is a relatively clear pattern of lead-lag relationships between the morning and the late afternoon/evening volatility components. Also, the lagged values of the midday volatility components are mostly insignificant in the equations for the mean values of the morning/late afternoon/evening volatility components. This means that the volatility in the beginning and in the end of the trading day is mainly determined by the volatility in the beginning and in the end of the previous trading day and not by the midday volatility in the previous trading day. The midday volatility also seems to have a dynamic behavior relatively independent from the beginning/end day volatility.

An exception is the lunch volatility which exhibits lead-lag relationships with both the morning/late afternoon/evening and late lunch/early afternoon volatility. Also, the coefficient for the morning volatility has usually a much higher value in comparison with the other coefficients, when it is significant. This suggests that the morning volatility is a quite important determinant of the next day volatility.

Next, the estimation results for the component MEM models for the realized power are presented in Table 5. In this case, we have two misspecifications. We cannot reject the hypothesis that the there is no autocorrelation up to lag 10 in the residuals of the morning and late lunch realized power. Note that previous authors (Forsberg and Ghysels (2004), Ding, Granger, and Engle (1993)) have noted that the realized power (absolute returns) are much more persistent than the realized volatility (daily squared returns). It is possible that a MEM specification is not able to capture this path-dependence in the realized power. Nevertheless, the autocorrealtions in the residuals seem to be relatively small. The restricted specifications show similar pattern of lead-lag relationships like that for the realized volatility and the realized range, although the distinction between the beginning/end day realized power and midday realized power is not as pronounced as before.

Finally, we consider two types of MEM models for the realized bi-power and the realized jump. First, separate MEM models for the components of the realized bi-power and the realized jump are estimated and the results are presented in Table 6 and Table 7, respectively without the interaction effect between the two measures. In all cases, we cannot reject the hypothesis that the fitted residuals are uncorrelated up to lag 10 at a 5% significance level. Also, the pattern of relatively independence between the lead-lag relationships of the morning/late afternoon/evening components and late lunch/early afternoon is preserved for both measures.

We also estimate MEM models for the realized bi-power and jumps, by including lagged values of the both realized power and realized jump components in order to test for interactions between the two measures. Table 8 and 9 present the estimation results for the component MEM models with the lead-lag interactions between the measures. There is no evidence for autocorellation in the fitted residuals up to lag 10 as indicated by the high p-values for the Ljung-Box statistics for both models reported in Table 8.1 and Table 9.1. As before, the pattern of lead-lag relationships between the intra-daily volatility components with the mixed data also holds. In addition, the morning/late afternoon/evening realized bi-power seems to lead the morning/late afternoon/evening realized jump, since the corresponding coefficients for the lagged realized jumps components are mostly insignificant. On the other hand, the coefficients for the lagged realized bi-power are often significant in the corresponding specifications for the realized jump components.

In summary, the main findings in this section are as follows. Firstly, the morning/late afternoon/evening volatility and late lunch/early afternoon volatility components seem to exhibit separate lead-lag behavior, such that the mixed lead-lag effects between both groups is not that pronounced as the lead-lag effects between the volatility components belonging to same group. However, the lagged values of the morning/late afternoon/evening volatility influence the next day late lunch/early afternoon volatility more often the lagged values of the lunch/early afternoon volatility influence the next day morning/late afternoon/evening volatility. Secondly, the morning volatility is an important determinant of the future volatility, since corresponding coefficients for its lagged value are rather high, when significant. Third, the realized bi-power seem to lead the realized jump.

6 Out-of-Sample Performance of the Component MEM models

A test for the economic significance of the intra-daily volatility seasonalities would be to check whether or not better out-of-sample predictions can be obtained by utilizing these patterns. We proceed by comparing forecasts of models which use only daily volatility measures to forecasts of the models, which employ the intra-daily components of the corresponding volatility measures. The comparison is done for prediction horizons with lengths between 1 and 20 business days (or up to 4 calendar weeks), since these time intervals are most relevant to practical tasks such as Value-at-Risk calculations and option pricing. The improvement or otherwise of the performance is measured by the relative change in the Root Mean Squared Error.

To fix ideas, we first discuss how to compare the daily and the component

MEM models for the realized volatility. For each forecasting horizon with length $f = 1, ..., 20$ and each trading day $t = 800, ..., 1209 - f$, the daily realized volatility MEM model is estimated using the observations up to the $t - 1$ observation, and on that basis a prediction for the realized volatility over the next f days p_t^f is formed. In a similar manner, a prediction for the next f days denoted by p_t^{*f}, is formed, using the realized volatility from the component MEM model. Then we compute the Root Mean Squared Errors (RMSEs) for each model.

Note that the RMSE criterion could be biased due to the overlapping forecasting horizons. To alleviate this problem, we divide the interval $[t, 1209]$ into f non-overlapping subsets $A_1, ..., A_f$ such that

$$A_1 = \{t, t + f, t + 2f, ...\}, \tag{32}$$

$$A_2 = \{t + 1, t + 1 + f, t + 1 + 2f, ...\}, \text{ and} \tag{33}$$

$$.....$$

$$A_f = \{t + f - 1, t + f - 1 + f, t + f - 1 + 2f, ...\}. \tag{34}$$

Each element of each subset points to the starting point of a prediction horizon with length f, such that the prediction horizons corresponding to the same subset do not overlap. Then for each subset $A_i, i = 1, ..., f$, we compute the RMSE of the two models for the forecasting horizons corresponding to that subset:

$$RMSE_i^f = \sqrt{\frac{1}{|A_i|} \cdot \sum_{j \in A_i} \left(p_j^f - RV_j - ... - RV_{j+f-1} \right)^2} \tag{35}$$

$$RMSE_i^{*f} = \sqrt{\frac{1}{|A_i|} \cdot \sum_{j \in A_i} \left(p_j^{*f} - RV_j - ... - RV_{j+f-1} \right)^2}. \tag{36}$$

As a next step, the Average Root Mean Squared Errors (ARMSEs) are calculated across all subsets $A_i, i = 1, ..., f$:

$$ARMSE^f = \frac{1}{f} \sum_{i=1}^{f} RMSE_i^f, \tag{37}$$

$$ARMSE^{*f} = \frac{1}{f} \sum_{i=1}^{f} RMSE_i^{*f}. \tag{38}$$

To gauge the improvement or otherwise of using the component realized volatility MEM model, we compute the values $ARMSE^{*f}/ARMSE^f$ for $f = 1, ..., 20$.

We proceed in the same way to compare the relative performance of the daily realized range versus the component realized range MEM models, and the daily realized power versus the component realized power MEM models. The results are shown in Table 10. As we can see, there is a substantial improvement in performance, which increases with the length of the forecasting horizon reaching almost 50-60% reduction of the RMSE for 20 days prediction horizons. On the other hand, improvement over very short term horizons, with a couple days length are negligible. One possible reason for this effect is that the intra-daily components capture more noise than the corresponding daily measure, affecting short term forecasts, and this noise is "smoothed out" when predictions for longer periods are formed.

Next, we analyze the realized bi-power and the realized jump. There are several natural ways by which we gauge the benefits of using component MEM models to form forecasts for these measures. Firstly, the benefits of using different modelling approaches for forecasting these two measures is assessed separately . For example, the realized bi-power is modelled using four different models: a daily MEM model, a daily MEM model with lagged values of the daily realized jumps, a component MEM model, and a component MEM model with the added lagged values of the realized jump components. We compare the performance of the first model, which serves as a benchmark, with the performance of the last three models. A similar approach is utilized for the realized jump.

We report the forecasting performance in Table 11. As it can be seen, using a component MEM approach significantly decreases the prediction error, such that the improvement increases with the length of the forecasting horizon for both the realized bi-power and realized jump. Modelling the joint daily realized bi-power and realized jump also results in a gain in forecasting performance, which is similar or slightly better than that of the respective component MEM models. The extended component MEM model for the realized bi-power with the lagged values of the realized jump components, and vice versa, does not perform better than the latter model.

Since the realized volatility is the sum of the realized bi-power and realized jump, predictions for it can be derived by adding the forecasts of the models for the realized bi-power and realized jump discussed in the previous section. More specifically, we consider four types of models. The first two are the combined daily models for the realized bi-power and realized jump without and with the interaction effects. The second two models are the combinations of the component MEM models for the realized bi-power and the realized jump, without and with the interaction affect. All these four models are compared with the benchmark daily MEM model for the realized volatility. The results are shown in Table 12.

The winners are models 2 and 3, where the latter model is slightly better. Model 4, which is a combination of the last two models performs worse, which is probably again due to the large number of parameters. The first model performs similarly to the benchmark model, which suggests that the interactions between the realized bi-power and realized jump are important.

To summarize the results, for all volatility measures, modelling separately the intra-daily components results in a significant gain in performance for longer forecasting horizons. In the case of the realized bi-power, realized jump and the realized volatility, we also use the approach of Andersen, Bollerslev and Diebold (2003) of modelling the interactions between the realized bi-power and realized jump. The resulting improvement in forecasts is comparable to that of our approach of using the intra-daily components. Combining the two approaches does not lead to any improvement and even results in a deterioration of the out-of-sample predictions.

7 Conclusion

In this paper we propose a new approach to volatility modelling which exploits the intra-daily volatility seasonalities. More specifically, it is a well established empirical fact that volatility follows approximately an inverted U-shaped pattern during the day. It is high in the morning, gradually decreasing, reaching a minimum at lunch time and then starting to increase again until the end of the trading day. We consider several measures related to the daily volatility and partite them into components, which correspond to the empirically observed intra-daily seasonalities. These components are modeled separately, while accounting for lead-lag relationship between them using the Multiplicative Error Model (MEM) of Engle (2002). Then we compare the forecasts for the daily volatility measures derived by summing the forecasts for the individual volatility components to forecasts obtained from models which use as inputs only the daily volatility measures.

In all cases, our approach leads to a significant improvement in the forecasting performance in terms of a sharp drop in the Root Mean Squared Error (RMSE). This improvement is negligible for short forecasting horizons, but gradually increases with the length of the horizon, reaching almost 50 percent reduction in RMSE for 20-days ahead predictions. These results show the potential importance of the intra-daily volatility seasonality to volatility forecasting. To our knowledge, this paper is the first one which assesses the economic significance of this widely observed pattern.

A natural extension of this work is to perform a similar analysis for other markets. Also, the modelling of the intra-daily volatility components can be done with models other than the MEM model. For example, Ghysels, Santa-Clara and Valkanov (2004) find that predicting the realized volatility in a Mixed Data Sampling (MIDAS) regression framework results in a significant improvement over the benchmark ARFIMA model of ABDL(2001,2003). Our approach of separately modeling the intra-daily realized volatility components can be directly applied in this framework, which could potentially lead to further increases in forecasting precision.

Finally, it is possible that similar results hold for the trading volume since in that case the same intra-daily seasonalities are observed. It is quite possible that better forecasts for measures of the trading activity, such as the number of transactions, can be obtained by separately modelling indicators which account for the trading during the different times of the day. Our preliminary analysis however (not reported in the paper) shows that the MEM model is inappropriate for this purpose, and so this remains a topic for future research.

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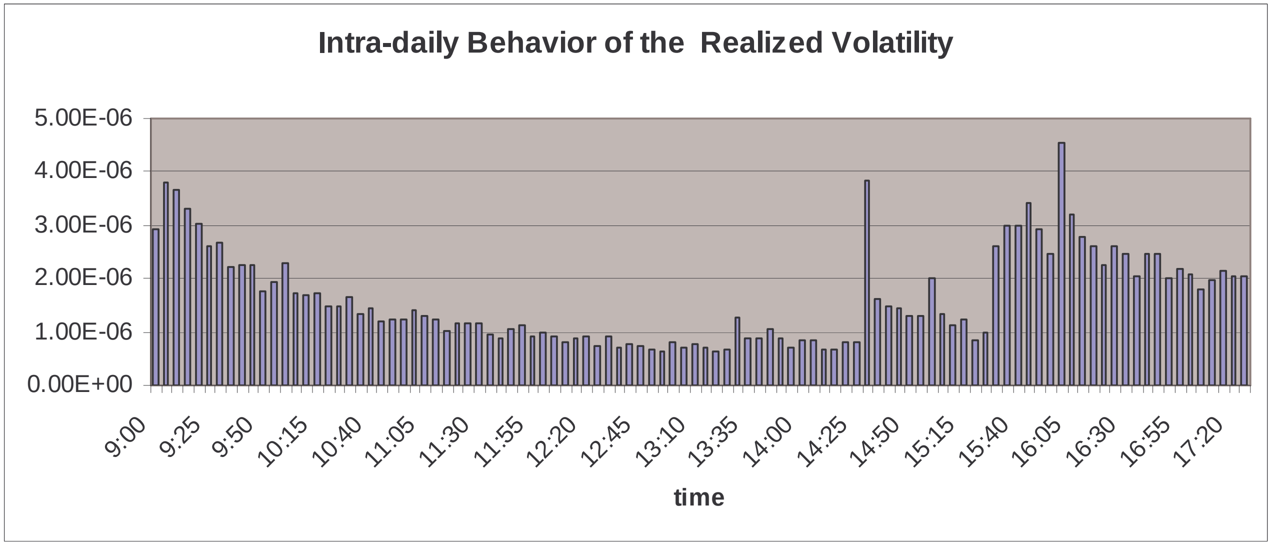
Graph 1



Intra-daily Behavior of the Realized Volatility
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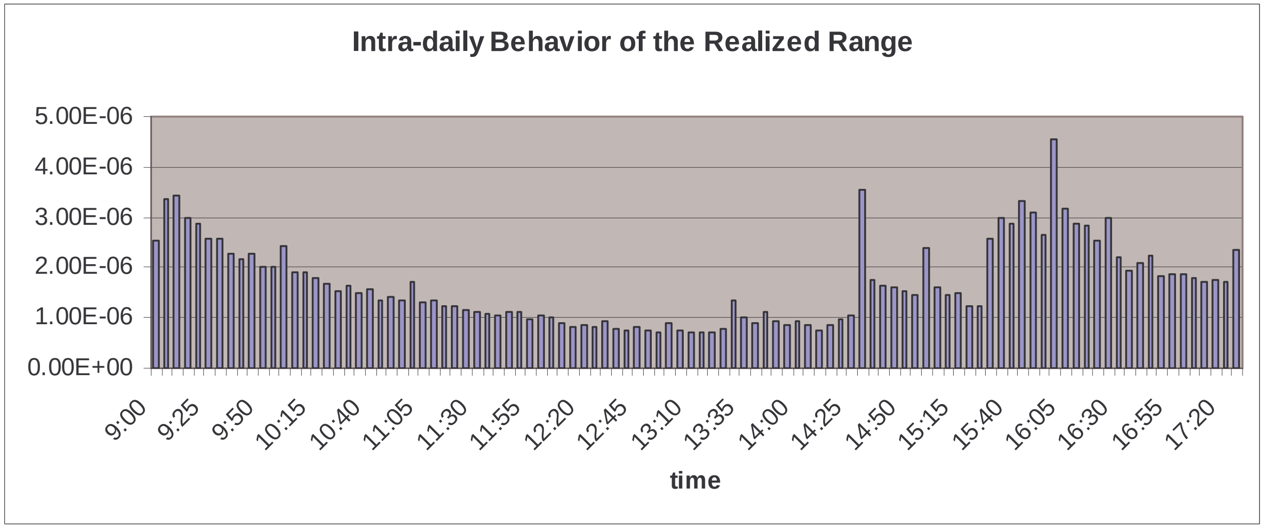
4.00E-06
3.00E-06
2.00E-06
1.00E-06
0.00E+00
9:00
9:25
9:50
10:15
10:40
11:05
11:30
11:55
12:20
12:45
13:10
13:35
14:00
14:25
14:50
15:15
15:40
16:05
16:30
16:55
17:20
time

Graph 2


Intra-daily Behavior of the Realized Range
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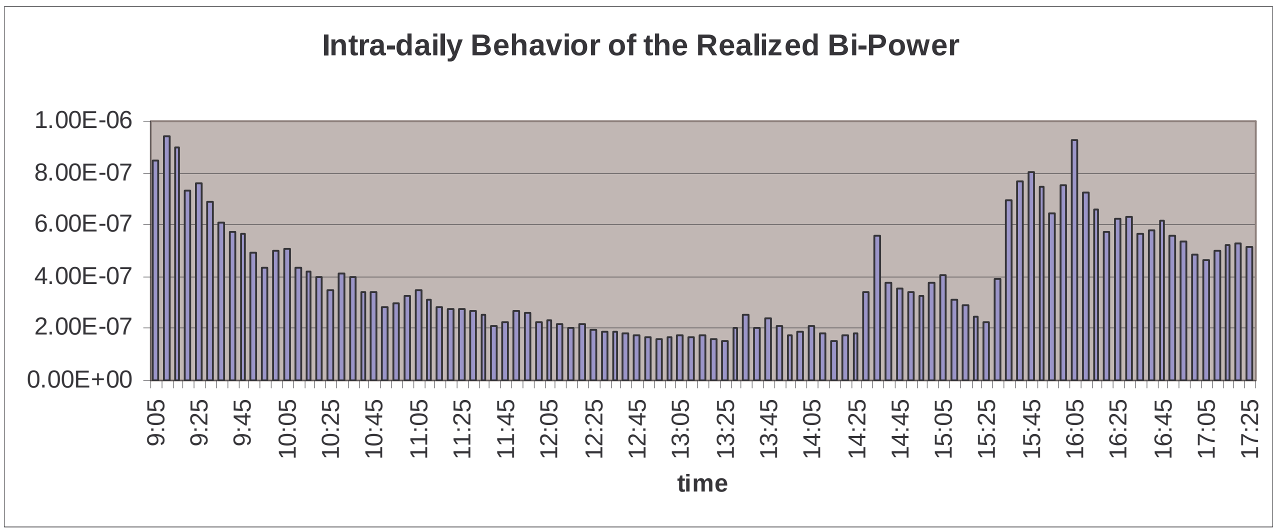
4.00E-06
3.00E-06
2.00E-06
1.00E-06
0.00E+00
9:00
9:25
9:50
10:15
10:40
11:05
11:30
11:55
12:20
12:45
13:10
13:35
14:00
14:25
14:50
15:15
15:40
16:05
16:30
16:55
17:20
time

Graph 3


Intra-daily Behavior of the Realized Bi-Power
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6.00E-07
4.00E-07
2.00E-07
0.00E+00
9:05
9:25
9:45
10:05
10:25
10:45
11:05
11:25
11:45
12:05
12:25
12:45
13:05
13:25
13:45
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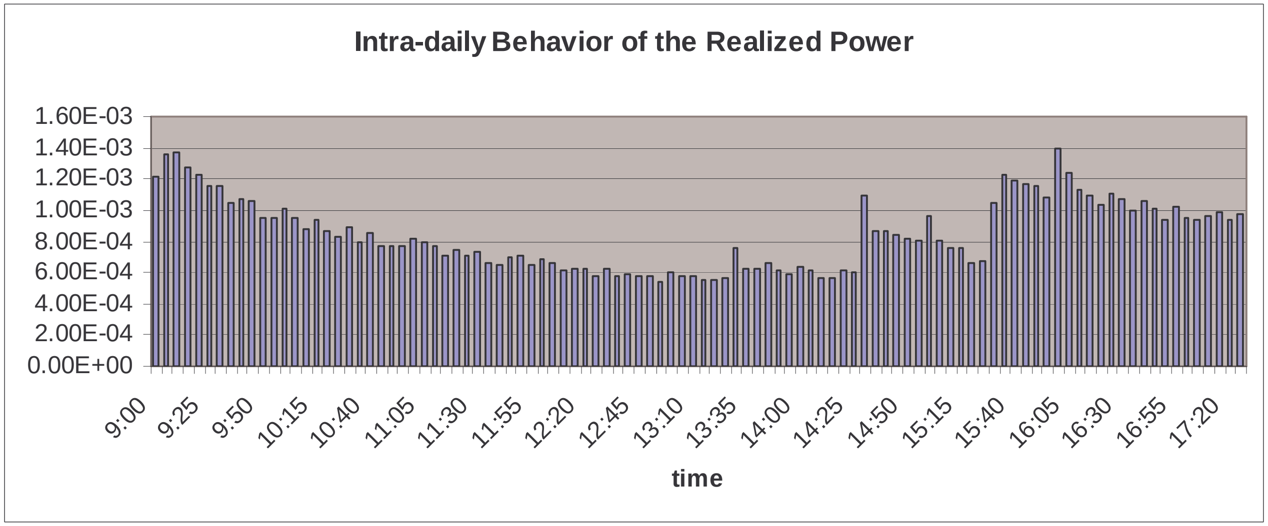
17:05
17:25
time

Graph 4



Intra-daily Behavior of the Realized Power
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1.20E-03
1.00E-03
8.00E-04
6.00E-04
4.00E-04
2.00E-04
0.00E+00
9:00
9:25
9:50
10:15
10:40
11:05
11:30
11:55
12:20
12:45
13:10
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16:55
17:20
time

Graph 5

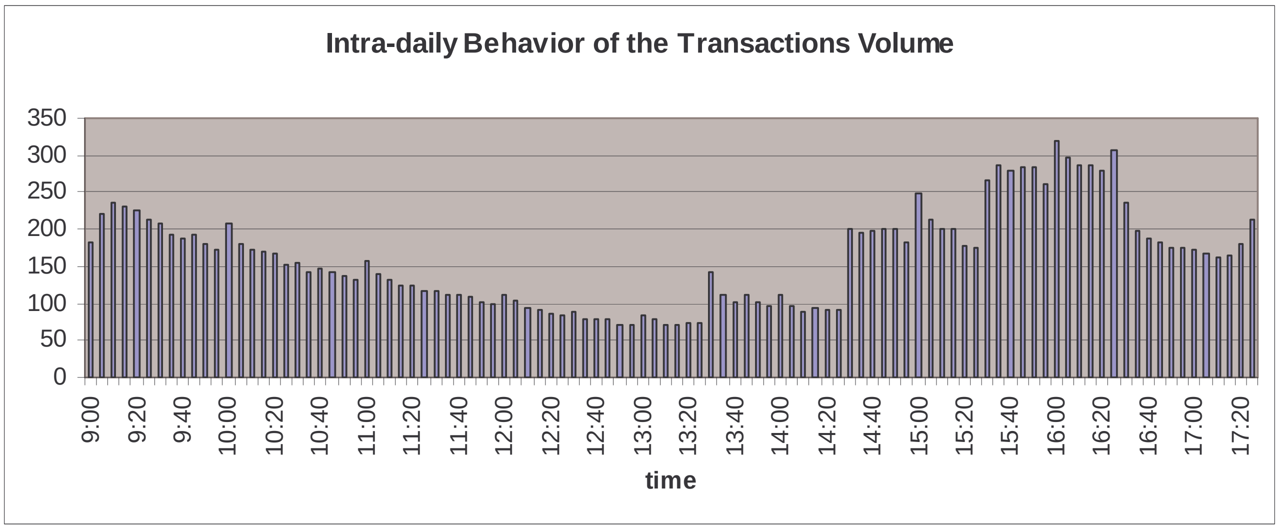
Intra-daily Behavior of the Transactions Volume
350
300
250
200
150
100
50
0
9:00
9:20
9:40
10:00
10:20
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Table 1

Summary Statistics for the Intra-daily Components of the Volatility Measures

Realized Volatility

	Morning[1]	Lunch	Lunch 1	Afternoon	Afternoon 1	Evening
Mean	1.99E-05	8.73E-06	1.03E-05	1.85E-05	3.54E-05	2.57E-05
Min	5.92E-07	1.31E-07	3.26E-07	4.21E-07	2.72E-07	3.21E-07
Max	0.000274	0.000174	0.000176	0.000342	0.001438	0.00061
St.Dev.	2.31E-05	1.22E-05	1.45E-05	3.09E-05	7.25E-05	4.27E-05

Realized Range

	Morning	Lunch	Lunch 1	Afternoon	Afternoon 1	Evening
Mean	2.03E-05	9.37E-06	1.16E-05	2.09E-05	3.65E-05	2.34E-05
Min	6.48E-07	2.52E-07	3.70E-07	5.61E-07	9.65E-07	3.10E-07
Max	0.000258	0.000156	0.000226	0.000408	0.001466	0.000505
St.Dev.	2.01E-05	1.18E-05	1.45E-05	2.73E-05	5.81E-05	3.51E-05

Realized Bi-Power

	Morning	Lunch	Lunch 1	Afternoon	Afternoon 1	Evening
Mean	1.20E-05	5.16E-06	5.97E-06	1.04E-05	2.08E-05	1.63E-05
Min	2.73E-07	5.09E-08	3.82E-08	1.96E-07	1.87E-07	1.18E-07
Max	0.000174	8.99E-05	0.000102	0.000219	0.001051	0.000485
St.Dev.	1.45E-05	7.34E-06	8.68E-06	1.51E-05	4.52E-05	3.03E-05

Realized Jump

	Morning	Lunch	Lunch 1	Afternoon	Afternoon 1	Evening
Mean	7.85E-06	3.57E-06	4.36E-06	8.14E-06	1.46E-05	9.45E-06
Min	1.75E-07	0	0	0	0	0
Max	0.000124	0.000101	9.19E-05	0.000263	0.000446	0.000139
St.Dev.	9.19E-06	5.85E-06	6.94E-06	1.89E-05	3.09E-05	1.49E-05

Realized Power

	Morning	Lunch	Lunch 1	Afternoon	Afternoon 1	Evening
Mean	0.010459	0.006928	0.007474	0.009931	0.013859	0.011848
Min	0.001943	0.000912	0.001304	0.001488	0.001566	0.001175
Max	0.043681	0.032886	0.035353	0.045913	0.117923	0.072279
St.Dev.	0.005593	0.004078	0.004239	0.005505	0.008919	0.007784

[1] Morning period refers to the time interval [9:00 AM; 12:30 AM]. Lunch period refers to the time interval [12:30 AM; 13:30 AM]. Lunch 1 period refers to the time interval [13:30 AM; 14:30 AM]. Afternoon period refers to the time interval [14:30 AM; 15:30 AM]. Afternoon 1 period refers to the time interval [15:30 AM; 16:30 AM]. Evening period refers to the time interval [16:30 AM; 17:30 AM].

Table 2

Summary Statistics for the Correlations between the Intra-daily Components of the Volatility Measures

Correlations between the intra-daily components of the realized volatility

	Morning[2]	Lunch	Lunch 1	Afternoon	Afternoon 1	Evening
Morning	1.00	0.63	0.47	0.36	0.49	0.66
Lunch	0.63	1.00	0.49	0.36	0.38	0.46
Lunch 1	0.47	0.49	1.00	0.31	0.30	0.35
Afternoon	0.36	0.36	0.31	1.00	0.51	0.41
Afternoon 1	0.49	0.38	0.30	0.51	1.00	0.64
Evening	0.66	0.46	0.35	0.41	0.64	1.00

Correlations between the intra-daily components of the realized range

	Morning	Lunch	Lunch 1	Afternoon	Afternoon 1	Evening
Morning	1.00	0.62	0.45	0.37	0.50	0.72
Lunch	0.62	1.00	0.51	0.33	0.39	0.46
Lunch 1	0.45	0.51	1.00	0.37	0.29	0.29
Afternoon	0.37	0.33	0.37	1.00	0.53	0.41
Afternoon 1	0.50	0.39	0.29	0.53	1.00	0.74
Evening	0.72	0.46	0.29	0.41	0.74	1.00

Correlations between the intra-daily components of the realized bi-power

	Morning	Lunch	Lunch 1	Afternoon	Afternoon 1	Evening
Morning	1.00	0.67	0.47	0.44	0.42	0.62
Lunch	0.67	1.00	0.50	0.47	0.36	0.45
Lunch 1	0.47	0.50	1.00	0.39	0.24	0.30
Afternoon	0.44	0.47	0.39	1.00	0.43	0.40
Afternoon 1	0.42	0.36	0.24	0.43	1.00	0.62
Evening	0.62	0.45	0.30	0.40	0.62	1.00

Correlations between the intra-daily components of the realized jump

	Morning	Lunch	Lunch 1	Afternoon	Afternoon 1	Evening
Morning	1.00	0.42	0.37	0.22	0.48	0.60
Lunch	0.42	1.00	0.30	0.18	0.29	0.34
Lunch 1	0.37	0.30	1.00	0.17	0.30	0.30
Afternoon	0.22	0.18	0.17	1.00	0.42	0.27
Afternoon 1	0.48	0.29	0.30	0.42	1.00	0.48
Evening	0.60	0.34	0.30	0.27	0.48	1.00

Correlations between the intra-daily components of the realized power

	Morning	Lunch	Lunch 1	Afternoon	Afternoon 1	Evening
Morning	1.00	0.75	0.60	0.44	0.72	0.78
Lunch	0.75	1.00	0.61	0.45	0.61	0.64
Lunch 1	0.60	0.61	1.00	0.47	0.52	0.52
Afternoon	0.44	0.45	0.47	1.00	0.58	0.47
Afternoon 1	0.72	0.61	0.52	0.58	1.00	0.76
Evening	0.78	0.64	0.52	0.47	0.76	1.00

[2] [2] Morning period refers to the time interval [9:00 AM; 12:30 AM]. Lunch period refers to the time interval [12:30 AM; 13:30 AM]. Lunch 1 period refers to the time interval [13:30 AM; 14:30 AM]. Afternoon period refers to the time interval [14:30 AM; 15:30 AM]. Afternoon 1 period refers to the time interval [15:30 AM; 16:30 AM]. Evening period refers to the time interval [16:30 AM; 17:30 AM].

Table 3

MEM Estimation for the Realized Volatility

	Morning Volatility		Lunch Volatility		Lunch 1 Volatility		Afternoon Volatility		Afternoon 1 Volatility		Evening Volatility	
Regressors	Full model	Restricted model	Full model	Restricted model	Full model	Restricted model	Full model	Restricted model	Full model	Restricted model	Full model	Restricted model
Intercept	1.19E-07 (7.9E-08)***	2.05E-07 (5.1E-08)***	4.85E-07 (1.3E-07)***	6.18E-07 (1.1E-07)***	4.61E-07 (1.5E-07)***	3.68E-07 (1.1E-07)***	9.95E-07 (3.5E-07)***	5.98E-07 (1.9E-07)***	4.28E-07 (1.9E-07)**	4.16E-07 (1.8E-07)**	-4.74E-08 (1.9E-08)**	-9.04E-08 (3.6E-08)**
CE	0.1592 (0.0276)***	0.1616 (0.0297)***	0.4044 (0.0825)***	0.3886 (0.0783)***	0.7875 (0.0603)***	0.7945 (0.0374)***	0.7032 (0.0629)***	0.7682 (0.0443)***	0.5774 (0.0829)***	0.5805 (0.0803)***	0.6809 (0.0457)***	0.6795 (0.0465)***
Morning	0.6744 (0.0307)***	0.6835 (0.0315)***	0.123294 (0.0298)***	0.1348 (0.0309)***	-0.0198 (0.0157)	-----------	-0.0564 (0.0433)	-----------	0.3759 (0.1049)***	0.3823 (0.1132)***	0.2424 (0.0441)***	0.2417 (0.0456)***
Lunch	0.02967 (0.0297)	-----------	0.048192 (0.0276)	0.0611 (0.0285)**	0.0814 (0.0300)***	0.1125 (0.0237)***	0.1257 (0.0797)	0.1454 (0.0592)**	0.0443 (0.1106)	-----------	0.0003 (0.0398)	
Lunch 1	0.01896 (0.0150)	-----------	-0.0020 (0.0102)	-----------	-0.0192 (0.0175)	-----------	0.1263 (0.0494)**	0.1225 (0.0515)**	-0.0186 (0.0302)	-----------	0.0098 (0.0146)	
Afternoon	-0.0004 (0.0063)	-----------	0.0140 (0.0094)	-----------	0.0507 (0.0196)***	0.0528 (0.0176)***	0.0722 (0.0236)***	0.0618 (0.0207)***	0.0055 (0.0194)	-----------	-0.01466 (0.0077)*	
Aft. 1	0.0270 (0.0097)***	0.030184 (0.0097)***	0.0117 (0.0084)	-----------	-0.0030 (0.0036)	-----------	0.0245 (0.0156)	-----------	0.1080 (0.0342)***	0.1114 (0.0334)***	0.0045 (0.0095)	
Evening	0.0754 (0.0167)***	0.0765 (0.0150)***	0.0533 (0.0168)***	0.0651 (0.0164)***	0.0323 (0.0162)**	-----------	0.0422 (0.0313)	-----------	0.1146 (0.0574)**	0.1136 (0.0564)**	0.1268 (0.0257)***	0.1293 (0.0262)***

Statistics for the Residuals

	Morning Volatility			Lunch Volatility			Lunch Volatility 1			Afternoon Volatility			Afternoon Volatility 1			Evening Volatility		
Lag	AC	Q-stat.	p-value	AC	Q-stat.	p-value	AC	Q-stat.	p-value	AC	Q-stat.	p-value	AC	Q-stat.	p-value	AC	Q-stat.	p-value
1	0.012	0.1783	0.673	0.009	0.009	0.1037	0.038	1.7331	0.188	-0.03	1.0936	0.296	-0.003	0.0114	0.915	-0.004	0.0179	0.894
2	-0.005	0.2055	0.902	-0.011	-0.011	0.2381	-0.023	2.3496	0.309	-0.013	1.306	0.52	-0.013	0.2283	0.892	-0.008	0.0978	0.952
3	-0.026	1.0377	0.792	-0.026	-0.026	1.0775	-0.064	7.3961	0.06	0.002	1.3112	0.726	-0.019	0.6594	0.883	-0.021	0.6553	0.884
4	-0.026	1.8572	0.762	0.007	0.007	1.1383	-0.018	7.8012	0.099	-0.014	1.5515	0.817	-0.009	0.7532	0.945	0.003	0.6664	0.955
5	-0.031	3.0459	0.693	-0.009	-0.01	1.2396	0.021	8.3365	0.139	0.049	4.4257	0.49	-0.003	0.7612	0.979	-0.034	2.0435	0.843
6	-0.003	3.0568	0.802	-0.007	-0.008	1.3067	0.035	9.8112	0.133	-0.002	4.4312	0.619	-0.022	1.3285	0.97	0.002	2.0477	0.915
7	0.01	3.169	0.869	-0.02	-0.02	1.7859	-0.028	10.78	0.149	0.013	4.6415	0.704	0.06	5.6911	0.576	0.059	6.3269	0.502
8	0.004	3.1887	0.922	0.014	0.014	2.0346	-0.01	10.91	0.207	0.059	8.8455	0.355	0.011	5.8308	0.666	0	6.3269	0.611
9	0.05	6.2144	0.718	0.011	0.01	2.1782	0.03	12.041	0.211	-0.003	8.8573	0.451	0	5.8308	0.757	0.019	6.7582	0.662
10	-0.024	6.9133	0.734	-0.005	-0.006	2.2111	0.066	17.302	0.068	-0.005	8.8851	0.543	-0.031	6.9688	0.728	0.007	6.8256	0.742

a) the upper table presents the restricted and restricted MEM models for the intra-daily volatility components. Restricted models are obtained by performing a Wald test and discarding regressors which are jointly insignificant at 5% level. The standard errors are reported in brackets and the coefficeints which are significant at 10%, 5 % and 1% level are marked with *,** and ***, respectively. With CE I denote the lagged value of the conditional expectation of the realized volatility.

b) the bottom table presents the autocorrelation coefficients, Q-statistics and the associated p-values for the residuals of the corresponding MEM models up to lag 10.

Table 4

MEM Estimation for the RR (Realized Range)

	Morning RR		Lunch RR		Lunch 1 RR		Afternoon RR		Afternoon 1 RR		Evening RR	
Regressors	Full model	Restricted model	Full model	Restricted model	Full model	Restricted model	Full model	Restricted model	Full model	Restricted model	Full model	Restricted model
Intercept	5.86E-07 (1.9E-07)***	6.61E-07 (2.2E-07)***	5.07E-07 (2.4E-07)***	5.13E-07 (2.3E-07)***	1.87E-07 (1.5E-07)***	1.82E-07 (8.5E-08)**	1.13E-06 (5.03E-07)***	1.28E-06 (6.5E-07)**	1.36E-06 (5.4E-07)**	1.39E-06 (5E-07)***	1.24E-07 (1.8E-08)***	3.18E-08 (2.5E-08)
CE	0.624087 (0.0306)***	0.6283 (0.0360)***	0.5186 (0.0668)***	0.5206 (0.0689)***	0.7815 (0.0373)***	0.7782 (0.0312)***	0.6560 (0.0560)***	0.5903 (0.0872)***	0.5148 (0.0810)***	0.5321 (0.0774)***	0.70784 (0.0378)***	0.7028 (0.0377)***
Morning	0.1527 (0.0305)***	0.1716 (0.0290)***	0.0420 (0.0298)*	0.0489 (0.0240)**	-0.0080 (0.0178)	-----------	-0.0431 (0.0470)	-----------	0.3199 (0.0903)***	0.4001 (0.0916)***	0.04301 (0.0152)***	0.0211 (0.0088)***
Lunch	0.0271 (0.0338)	-----------	0.0457 (0.0264)*	-----------	0.0852 (0.0288)***	0.0866 (0.0210)***	0.0746 (0.090)	-----------	0.0661 (0.0957)	-----------	-0.0151 (0.0131)	-----------
Lunch 1	0.0079 (0.0221)	-----------	-0.0147 (0.0163)	-----------	0.0122 (0.0182)	-----------	0.1053 (0.0468)**	0.0934 (0.0557)	0.0484 (0.0536)	-----------	-0.0097 (0.0089)	-----------
Afternoon	0.0110 (0.0090)	-----------	0.0583 (0.0147)***	0.0645 (0.0125)***	0.0659 (0.0176)***	0.0711 (0.0161)***	0.07633 (0.02289)**	0.0923 (0.0336)***	0.0220 (0.0232)	-----------	-0.0025 (0.0017)	-0.0032 (0.0018)*
Aft. 1	0.0377 (0.0114)***	0.0480 (0.0103)***	0.0179 (0.0117)	-----------	0.0075 (0.0071)	-----------	0.1329 (0.0517)**	-----------	0.1689 (0.0480)***	0.2058 (0.0455)***	-0.0058 (0.0051)	-----------
Evening	0.0879 (0.0181)***	0.0732 (0.0163)***	0.0464 (0.0157)***	0.0564 (0.0141)***	-0.0015 (0.0106)	-----------	-0.05451 (0.0324)*	-----------	0.0862 (0.0425)**	0.1136 (0.0564)**	0.2738 (0.0398)***	0.2845 (0.0400)***

Statistics for the Residuals

	Morning RR			Lunch RR			Lunch RR 1			Afternoon RR			Afternoon RR 1			Evening RR		
Lag	AC	Q-stat.	p-value	AC	Q-stat.	p-value	AC	Q-stat.	p-value	AC	Q-stat.	p-value	AC	Q-stat.	p-value	AC	Q-stat.	p-value
1	-0.032	1.2466	0.264	0.005	0.0312	0.860	0.038	1.73	0.188	0.002	0.0054	0.941	-0.007	0.0578	0.81	0.021	0.5234	0.469
2	0.034	2.6675	0.263	-0.026	0.8388	0.657	-0.05	4.7967	0.091	-0.027	0.8761	0.645	-0.028	0.9767	0.614	-0.046	3.0471	0.218
3	-0.009	2.7694	0.429	-0.001	0.8400	0.840	-0.087	14.067	0.003	-0.006	0.9249	0.819	-0.001	0.9787	0.806	-0.042	5.2032	0.158
4	0.033	4.0765	0.396	0.061	5.4071	0.248	0.024	14.774	0.005	0.025	1.7112	0.789	-0.016	1.309	0.86	-0.001	5.2052	0.267
5	-0.007	4.1345	0.53	-0.015	5.6727	0.339	0.053	18.209	0.003	0.090	11.577	0.041	0.007	1.3655	0.928	-0.016	5.5179	0.356
6	-0.015	4.4039	0.622	-0.035	7.1538	0.307	0.001	18.21	0.006	-0.018	11.964	0.063	-0.017	1.7273	0.943	-0.015	5.7951	0.447
7	-0.058	8.5261	0.288	0.016	7.4493	0.384	-0.045	20.706	0.004	0.055	15.639	0.029	0.065	6.9292	0.436	0.077	12.942	0.074
8	0.011	8.6749	0.37	0.012	7.6132	0.472	0.006	20.752	0.008	0.066	20.940	0.007	0.012	7.1093	0.525	-0.017	13.289	0.102
9	0.038	10.429	0.317	0.044	9.9816	0.352	0.044	23.159	0.006	0.041	22.966	0.006	0.017	7.4508	0.59	0.015	13.577	0.138
10	-0.021	10.984	0.359	-0.003	9.9898	0.441	0.11	37.87	0	0.078	30.367	0.001	-0.04	9.4014	0.494	-0.01	13.691	0.188

c) the upper table presents the restricted and restricted MEM models for the intra-daily volatility components. Restricted models are obtained by performing a Wald test and discarding regressors which are jointly insignificant at 5% level. The standard errors are reported in brackets and the coefficients which are significant at 10%, 5 % and 1% level are marked with *,** and ***, respectively. With CE I denote the lagged value of the conditional expectation of the realized range.

d) the bottom table presents the autocorrelation coefficients, Q-statistics and the associated p-values for the residuals of the corresponding MEM models up to lag 10.

Table 5

MEM Estimation for the RP (Realized Power)

	Morning RP		Lunch RP		Lunch 1 RP		Afternoon RP		Afternoon 1 RP		Evening RP	
Regressors	Full model	Restricted model	Full model	Restricted model	Full model	Restricted model	Full model	Restricted model	Full model	Restricted model	Full model	Restricted model
Intercept	9.02E-05 (8.7E-05)	0.0001 (6.0E-05)***	0.0004 (0.0001)***	0.0005 (0.0001)***	0.0004 (0.0001)***	0.0004 (0.0001)***	0.0005 (0.0002)***	0.0004 (0.0001)***	0.0005 (0.0002)***	0.0006 (0.0001)***	5.03E-05 (9.3E-05)	9.11E-05 (9E-05)
CE	0.4786 (0.0388)***	0.6064 (0.0316)***	0.4440 (0.0678)***	0.4284 (0.0669)***	0.7017 (0.0526)***	0.7259 (0.0470)***	0.7378 (0.0401)***	0.7770 (0.0338)***	0.5726 (0.0681)***	0.5622 (0.0676)***	0.6442 (0.0417)***	0.6357 (0.0431)***
Morning	0.3071 (0.0327)***	0.2269 (0.0284)***	0.1489 (0.0352)***	0.1576 (0.0346)**	0.0088 (0.0267)	-----------	-0.0763 (0.0328)**	-0.0374 (0.0215)*	0.2400 (0.0644)***	0.2370 (0.0661)***	0.2632 (0.0428)***	0.2515 (0.0420)***
Lunch	0.0489 (0.0272)*	0.0536 (0.0219)**	0.0672 (0.030)**	0.0768 (0.0308)**	0.0643 (0.0306)**	0.0635 (0.0270)**	0.0420 (0.0384)	-----------	-0.0042 (0.0505)	-----------	-0.0441 (0.0317)	-----------
Lunch 1	0.0120 (0.0183)	-----------	0.0269 (0.0193)	0.0398 (0.0176)**	-0.0063 (0.0241)	-----------	0.1002 (0.0306)**	0.1033 (0.0277)***	0.0097 (0.0339)	-----------	0.0170 (0.0241)	-----------
Afternoon	0.0051 (0.0139)	-----------	0.0140 (0.0152)	-----------	0.0786 (0.0189)***	0.0752 (0.0165)***	0.1098 (0.0230)***	0.0950 (0.0209)***	0.0152 (0.0226)	-----------	-0.0336 (0.0148)**	-0.0368 (0.0128)***
Aft. 1	0.0147 (0.0146)	-----------	0.0184 (0.0161)	-----------	-0.0010 (0.0149)	-----------	0.0318 (0.020)*	0.0390 (0.0167)**	0.1005 (0.0295)***	0.1164 (0.0287)***	0.0395 (0.0182)**	0.0390 (0.0185)**
Evening	0.1228 (0.0193)***	0.1070 (0.0158)***	0.0720 (0.0214)***	0.0855 (0.0196)***	0.0476 (0.0225)**	0.0471 (0.0167)***	0.0292 (0.0231)	-----------	0.1150 (0.0378)***	0.1191 (0.0380)***	0.1144 (0.0264)***	0.1180 (0.0269)***

Statistics for the Residuals

	Morning RP			Lunch RP			Lunch RP 1			Afternoon RP			Afternoon RP 1			Evening RP		
Lag	AC	Q-stat.	p-value	AC	Q-stat.	p-value	AC	Q-stat.	p-value	AC	Q-stat.	p-value	AC	Q-stat.	p-value	AC	Q-stat.	p-value
1	-0.071	6.1692	0.013	0.023	0.65	0.42	0.041	2.0167	0.156	-0.034	1.3821	0.24	0	0.0002	0.99	0.002	0.0056	0.94
2	-0.052	9.4467	0.009	-0.032	1.8631	0.394	-0.031	3.1738	0.205	-0.019	1.8413	0.398	-0.019	0.4218	0.81	-0.012	0.1871	0.911
3	-0.054	13.01	0.005	-0.035	3.3159	0.345	-0.088	12.502	0.006	0.014	2.08	0.556	-0.026	1.2516	0.741	-0.039	2.0101	0.57
4	-0.019	13.454	0.009	-0.012	3.5021	0.478	-0.023	13.157	0.011	-0.014	2.3048	0.68	-0.007	1.3075	0.86	0.015	2.2816	0.684
5	-0.015	13.739	0.017	0.002	3.5095	0.622	0.073	19.589	0.001	0.082	10.487	0.063	0.04	3.2852	0.656	-0.012	2.4649	0.782
6	-0.013	13.932	0.03	-0.016	3.8149	0.702	0.023	20.228	0.003	-0.034	11.887	0.065	-0.022	3.8544	0.696	0.006	2.5154	0.867
7	0.041	16.001	0.025	-0.023	4.4801	0.723	-0.036	21.79	0.003	-0.001	11.888	0.104	0.055	7.5432	0.375	0.002	2.5208	0.926
8	0.005	16.037	0.042	0.023	5.14	0.743	-0.021	22.322	0.004	0.01	12.006	0.151	0.018	7.9346	0.44	-0.015	2.8109	0.946
9	0.029	17.032	0.048	0.033	6.4624	0.693	0.044	24.661	0.003	-0.007	12.063	0.21	0.014	8.168	0.517	0.059	7.056	0.631
10	-0.004	17.056	0.073	0.012	6.6269	0.76	0.087	33.825	0	0.022	12.63	0.245	-0.01	8.2783	0.602	0.012	7.2379	0.703

e) the upper table presents the restricted and restricted MEM models for the intra-daily volatility components. Restricted models are obtained by performing a Wald test and discarding regressors which are jointly insignificant at 5% level. The standard errors are reported in brackets and the coefficeints which are significant at 10%, 5 % and 1% level are marked with *,** and ***, respectively. With CE I denote the lagged value of the conditional expectation of the realized power.

f) the bottom table presents the autocorrelation coefficients, Q-statistics and the associated p-values for the residuals of the corresponding MEM models up to lag 10.

Table 6

MEM Estimation for the RBP (Realized Bi-Power)

	Morning RBP		Lunch RBP		Lunch 1 RBP		Afternoon RBP		Afternoon 1 RBP		Evening RBP	
Regressors	Full model	Restricted model	Full model	Restricted model	Full model	Restricted model	Full model	Restricted model	Full model	Restricted model	Full model	Restricted model
Intercept	4.20E-08 (3.38E-08)	7.58E-08 (4.2E-08)**	1.53E-07 (2.2E-08)***	1.91E-07 (2.9E-08)***	2.64E-07 (6.1E-08)***	2.45E-07 (8.9E-08)***	4.95E-07 (1.4E-07)***	4.56E-07 (1.1E-07)***	3.56E-07 (1.29E-07)***	3.86E-07 (6.9E-08)***	6.82E-08 (1.8E-08)***	6.12E-09 (3.5E-08)
CE	0.6538 (0.031)***	0.6536 (0.0337)***	0.4620 (0.0624)***	0.4623 (0.0532)***	0.7606 (0.0396)***	0.7809 (0.0357)***	0.7498 (0.0461)***	0.75662 (0.0370)***	0.5503 (0.0896)***	0.5520 (0.080)***	0.6363 (0.0549)***	0.6342 (0.0553)***
Morning	0.1893 (0.0278)***	0.1974 (0.0317)***	0.1016 (0.0242)***	0.1076 (0.0240)***	0.0074 (0.0188)	-----------	-0.0421 (0.0306)**	-----------	0.3385 (0.1150)***	0.350 (0.1222)***	0.3299 (0.0648)***	0.3204 (0.0651)***
Lunch	0.0591 (0.0321)*	0.0705 (0.0294)**	0.0834 (0.0364)**	0.09144 (0.0362)**	0.0402 (0.0316)	-----------	0.0612 (0.0520)	-----------	0.0244 (0.1246)	-----------	-0.0326 (0.0409)	-----------
Lunch 1	0.0203 (0.0154)	-----------	-0.00512 (0.0103)	-----------	-0.0063 (0.0241)	-----------	0.0734 (0.0322)**	0.0725 (0.0309)**	0.0254 (0.0530)	-----------	-0.0079 (0.0184)	-----------
Afternoon	-0.0026 (-0.0026)	-----------	0.0010 (0.0080)	-----------	0.0483 (0.0133)***	0.0446 (0.0124)***	0.0898 (0.0225)***	0.0990 (0.0250)***	-0.0029 (0.0258)	-----------	-0.0122 (0.0113)	-----------
Aft. 1	0.0104 (0.0065)	-----------	0.0084 (0.0075)	-----------	-0.0044 (0.0030)	-----------	0.0126 (0.0116)*	0.0390 (0.0167)**	0.1553 (0.052)***	0.1573 (0.0486)***	0.0060 (0.0010)	-----------
evening	0.0826 (0.0155)***	0.0904 (0.0148)***	0.0550 (0.0145)***	0.0606 (0.0138)***	0.0360 (0.0153)**	0.0402 (0.0090)***	0.0427 (0.0203)**	0.0394 (0.0130)***	0.1026 (0.0461)**	0.1036 (0.0439)***	0.1177 (0.0281)***	0.1162 (0.0277)***

Statistics for the Residuals

	Morning RBP			Lunch RBP			Lunch RBP 1			Afternoon RBP			Afternoon RBP 1			Evening RBP		
Lag	AC	Q-stat.	p-value	AC	Q-stat.	p-value	AC	Q-stat.	p-value	AC	Q-stat.	p-value	AC	Q-stat.	p-value	AC	Q-stat.	p-value
1	0.022	0.5873	0.443	0.009	0.0945	0.759	0.033	1.3221	0.25	-0.018	0.3764	0.54	-0.003	0.0083	0.927	0.003	0.0138	0.907
2	-0.024	1.2634	0.532	-0.021	0.618	0.734	-0.025	2.0506	0.359	-0.024	1.0969	0.578	-0.015	0.2821	0.868	-0.029	1.0332	0.597
3	-0.021	1.7818	0.619	-0.024	1.2917	0.731	-0.054	5.535	0.137	-0.007	1.1637	0.762	-0.025	1.0123	0.798	-0.022	1.609	0.657
4	-0.039	3.5858	0.465	-0.02	1.7815	0.776	-0.021	6.078	0.193	-0.02	1.6647	0.797	-0.02	1.4797	0.83	0.032	2.8299	0.587
5	-0.034	5.0214	0.413	0.015	2.0419	0.843	0.027	6.9845	0.222	0.032	2.9382	0.71	-0.008	1.5502	0.907	-0.033	4.1364	0.53
6	-0.025	5.7613	0.45	0	2.042	0.916	0.034	8.3639	0.213	-0.034	4.3151	0.634	-0.016	1.868	0.931	-0.016	4.4658	0.614
7	0.044	8.0644	0.327	-0.017	2.4139	0.933	-0.007	8.4267	0.296	0.039	6.1565	0.522	0.041	3.932	0.788	0.063	9.3278	0.23
8	0.01	8.1779	0.416	0.026	3.2565	0.917	-0.011	8.5651	0.38	0.03	7.2891	0.506	0.013	4.1292	0.845	-0.01	9.4432	0.306
9	0.026	9.0267	0.435	0.033	4.5589	0.871	0.034	9.9468	0.355	0.018	7.6724	0.567	-0.017	4.465	0.878	0.045	11.86	0.221
10	-0.017	9.3848	0.496	0.008	4.6404	0.914	0.074	16.667	0.082	0.017	8.0052	0.628	-0.031	5.6629	0.843	0.01	11.989	0.286

g) the upper table presents the restricted and restricted MEM models for the intra-daily volatility components. Restricted models are obtained by performing a Wald test and discarding regressors which are jointly insignificant at 5% level. The standard errors are reported in brackets and the coefficeints which are significant at 10%, 5 % and 1% level are marked with *,** and ***, respectively. With CE I denote the lagged value of the conditional expectation of the realized bi-power.

h) the bottom table presents the autocorrelation coefficients, Q-statistics and the associated p-values for the residuals of the corresponding MEM models up to lag 10.

Table 7

MEM Estimation for the RJ (Realized Jump)

Regressors	Morning RJ Full model	Morning RJ Restricted model	Lunch RJ Full model	Lunch RJ Restricted model	Lunch 1 RJ Full model	Lunch 1 RJ Restricted model	Afternoon RJ Full model	Afternoon RJ Restricted model	Afternoon 1 RJ Full model	Afternoon 1 RJ Restricted model	Evening RJ Full model	Evening RJ Restricted model
Intercept	9.17E-08 (4.6E-08)**	1.20E-07 (4.97E-08)**	2.45E-07 (6.2E-08)***	2.67E-07 (5.5E-08)***	2.03E-07 (6.9E-08)***	1.78E-07 (1.5E-07)	3.56E-07 (1.21E-07)***	8.17E-08 (1.7E-07)	8.24E-08 (5.3E-08)	2.22E-09 (6.6E-08)	-6.11E-08 (2.1E-08)***	-4.50E-08 (3.8E-08)
CE	0.7406 (0.0273)***	0.7465 (0.0269)***	0.4486 (0.0764)***	0.4306 (0.0652)***	0.8060 (0.0398)***	0.8119 (0.0388)***	0.7456 (0.0589)***	0.9016 (0.0300)***	0.6408 (0.0640)***	0.6695 (0.0549)***	0.8184 (0.0239)***	0.8142 (0.0250)***
Morning	0.1100 (0.0256)***	0.1096 (0.0248)***	0.1016 (0.1051)***	0.1186 (0.0264)***	-0.0266 (0.0191)	-----------	-0.0394 (0.0509)	-----------	0.3859 (0.0976)***	0.4134 (0.0907)***	0.0866 (0.0203)***	0.0884 (0.0203)***
Lunch	0.0091 (0.0190)	0.0705 (0.0294)**	0.0317 (0.0251)	0.05264 (0.0261)**	0.0964 (0.0335)***	0.0939 (0.0268)***	0.0870 (0.0739)	-----------	0.0678 (0.0713)	-----------	0.0138 (0.0209)	-----------
Lunch 1	0.0093 (0.0134)	-----------	0.0184 (0.0143)	-----------	0.0002 (0.0140)	-----------	0.1288 (0.0427)***	0.097983 (0.0309)**	-0.0247 (0.0184)	-----------	0.0120 (0.0092)	-----------
Afternoon	0.0027 (0.0045)	-----------	0.0241 (0.0111)**	0.0301 (0.0116)***	0.0385 (0.0149)***	0.0407 (0.0153)***	0.0384 (0.0187)**	0.0990 (0.0250)***	-0.0097 (0.0090)	-----------	-0.0081 (0.0052)	-----------
Aft. 1	0.0362 (0.0093)***	0.0386 (0.0084)***	0.0129 (0.0079)	-----------	0.0102 (0.0065)	-----------	0.0284 (0.0194)	-----------	0.0750 (0.0247)***	0.0978 (0.030)***	0.0202 (0.0086)**	-----------
Evening	0.0535 (0.0129)***	0.0523 (0.0127)***	0.0365 (0.0164)**	0.0450 (0.0138)***	0.0078 (0.0098)	-----------	0.0467 (0.0397)	-----------	0.0845 (0.0474)**	-----------	0.0803 (0.0177)***	0.0852 (0.0175)***

Statistics for the Residuals

Lag	Morning RJ AC	Morning RJ Q-stat.	Morning RJ p-value	Lunch RJ AC	Lunch RJ Q-stat.	Lunch RJ p-value	Lunch RJ 1 AC	Lunch RJ 1 Q-stat.	Lunch RJ 1 p-value	Afternoon RJ AC	Afternoon RJ Q-stat.	Afternoon RJ p-value	Afternoon RJ 1 AC	Afternoon RJ 1 Q-stat.	Afternoon RJ 1 p-value	Evening RJ AC	Evening RJ Q-stat.	Evening RJ p-value
1	0.016	0.3289	0.566	0.015	0.2613	0.609	0.015	0.2672	0.605	-0.019	0.4438	0.505	-0.003	0.0116	0.914	0.005	0.0311	0.86
2	-0.017	0.687	0.709	-0.01	0.3789	0.827	-0.008	0.3429	0.842	0.002	0.4471	0.8	-0.004	0.0323	0.984	-0.004	0.0516	0.975
3	-0.044	2.9933	0.393	-0.032	1.6534	0.647	-0.057	4.2258	0.238	-0.021	0.9704	0.808	-0.009	0.1257	0.989	-0.003	0.0645	0.996
4	-0.036	4.5722	0.334	0.016	1.9749	0.74	-0.021	4.7454	0.314	-0.001	0.9707	0.914	0.012	0.315	0.989	-0.029	1.0904	0.896
5	0.028	5.5157	0.356	-0.035	3.4419	0.632	0.018	5.1303	0.4	0.027	1.872	0.867	-0.007	0.3738	0.996	-0.015	1.3737	0.927
6	0.011	5.6628	0.462	0.017	3.8107	0.702	0.017	5.4945	0.482	0.021	2.426	0.877	-0.003	0.3886	0.999	0.004	1.3919	0.966
7	0.023	6.2957	0.506	0	3.8109	0.801	-0.029	6.4858	0.484	-0.011	2.5604	0.922	0.063	5.2844	0.625	0.04	3.3802	0.848
8	-0.038	8.0946	0.424	-0.011	3.9659	0.86	-0.008	6.5554	0.585	0.054	6.0817	0.638	0.007	5.3501	0.72	0.02	3.8763	0.868
9	0.042	10.217	0.333	-0.019	4.4282	0.881	0.016	6.8742	0.65	-0.016	6.3924	0.7	0.019	5.8042	0.759	-0.027	4.778	0.853
10	-0.034	11.597	0.313	-0.02	4.9405	0.895	0.033	8.2056	0.609	-0.012	6.5698	0.765	-0.012	5.9738	0.817	0.012	4.9559	0.894

i) the upper table presents the restricted and restricted MEM models for the intra-daily volatility components. Restricted models are obtained by performing a Wald test and discarding regressors which are jointly insignificant at 5% level. The standard errors are reported in brackets and the coefficeints which are significant at 10%, 5 % and 1% level are marked with *,** and ***, respectively. With CE I denote the lagged value of the conditional expectation of the realized jump.

j) the bottom table presents the autocorrelation coefficients, Q-statistics and the associated p-values for the residuals of the corresponding MEM models up to lag 10.

Table 8

MEM Estimation for the RBP (Realized Bi-Power) with Lagged Values of the Realized Jump

	Morning RBP		Lunch RBP		Lunch 1 RBP		Afternoon RBP		Afternoon 1 RBP		Evening RBP	
Regressors	Full model	Restricted model	Full model	Restricted model	Full model	Restricted model	Full model	Restricted model	Full model	Restricted model	Full model	Restricted model
Intercept	1.56E-08 (2.2E-08)	4.30E-08 (4.39E-08)	1.37E-07 (2.2E-08)***	1.65E-07 (2.7E-08)***	2.86E-07 (8.0E-08)***	2.45E-07 (8.9-08)***	5.02E-07 (1.7E-07)***	4.92E-07 (1.1E-07)	2.28E-07 (1.18E-07)*	3.86E-07 (6.9E-08)***	7.76E-08 (2.1E-08)***	5.67E-09 (2.2E-08)
CE	0.6538 (0.0332)***	0.6550 (0.0324)***	0.4777 (0.0624)***	0.4711 (0.0558)***	0.7645 (0.0545)***	0.7809 (0.0357)***	0.7043 (0.0438)***	0.7089 (0.0382)***	0.5694 (0.0796)***	0.5520 (0.0800)***	0.6189 (0.0558)***	0.6300 (0.0585)***
Morning	0.1861 (0.0297)***	0.1842 (0.0294)***	0.0537 (0.0301)*	0.0672 (0.0288)**	0.0144 (0.0307)	-----------	-0.0505 (0.0501)	-----------	0.2209 (0.1400)	0.3501 (0.1222)***	0.3392 (0.0802)***	0.3092 (0.0665)***
Lunch	0.0765 (0.0372)**	0.0652 (0.0293)**	0.0870 (0.0344)**	0.0934 (0.0372)**	0.0567 (0.0338)*	-----------	-0.0692 (0.0562)	-----------	-0.0311 (0.1469)	-----------	-0.0242 (0.0500)	-----------
Lunch 1	0.0208 (0.0207)	-----------	0.0096 (0.0133)	-----------	0.0012 (0.0204)	-----------	0.0397 (0.0463)	-----------	0.0145 (0.0593)	-----------	-0.0056 (0.0231)	-----------
Afternoon	-0.0081 (0.0142)	-----------	0.0110 (0.0118)	-----------	0.0267 (0.0157)*	0.0446 (0.0124)***	0.0640 (0.0275)**	0.0800 (0.0237)***	-0.0400 (0.0420)	-----------	-0.0008 (0.0180)	-----------
Aft. 1	-0.0069 (0.0125)	-----------	0.0117 (0.0095)	-----------	-0.0042 (0.0077)	-----------	0.0107 (0.0144)	-----------	0.1421 (0.0555)**	0.1573 (0.0486)***	2.90E-06 (0.0132)	-----------
Evening	0.06171 (0.0168)***	0.0732 (0.0154)***	0.0489 (0.0158)***	0.060822 (0.0140)***	0.0380 (0.0194)*	0.0402 (0.0090)***	0.0495 (0.0286)*	0.0328 (0.0142)**	0.1093 (0.0508)**	0.1036 (0.0439)**	0.0820 (0.0284)***	0.0823 (0.0279)***
MorningJ	-0.0053 (0.0287)	-----------	0.0690 (0.0315)**	0.0686 (0.0313)**	-0.0193 (0.0402)	-----------	0.0464 (0.0742)	-----------	0.1963 (0.1287)	-----------	-0.0051 (0.0580)	-----------
LunchJ	-0.0256 (0.0247)	-----------	-0.0374 (0.0165)**	-0.0281 (0.0178)	0.0165 (0.0411)	-----------	0.2587 (0.0884)***	0.1953 (0.0705)***	0.0771 (0.1660)	-----------	-0.0249 (0.0473)	-----------
Lunch 1 J	0.0071 (0.0251)	-----------	-0.0154 (0.0094)*	-----------	-0.0244 (0.0179)	-----------	0.0965 (0.0448)**	0.1132 (0.0357)***	0.0065 (0.0433)	-----------	-0.0078 (0.0163)	-----------
AfternoonJ	0.0085 (0.0148)	-----------	-0.0012 (0.010)	-----------	0.0278 (0.0213)	-----------	0.0053 (0.0223)	-----------	0.0456 (0.0468)	-----------	-0.0103 (0.0137	-----------
Aft.1 J	0.0390 (0.0093)**	0.0343 (0.0142)**	-0.0062 (0.0115)	-----------	-0.0022 (0.0132)	-----------	0.0023 (0.0210)	-----------	-0.0031 (0.0427)	-----------	0.0130 (0.0192)	-----------
EveningJ	0.0217 (0.0191)	-----------	0.0154 (0.0170)	-----------	-0.0126 (0.0166)	-----------	-0.0147 (0.0310)	-----------	-0.0186 (0.0771)	-----------	0.0756 (0.0359)***	0.0754 (0.0366)**

a) the upper table presents the restricted and restricted MEM models for the intra-daily volatility components. Restricted models are obtained by performing a Wald test and discarding regressors which are jointly insignificant at 5% level. The standard errors are reported in brackets and the coefficeints which are significant at 10%, 5 % and 1% level are marked with *,** and ***, respectively. With CE I denote the lagged value of the conditional expectation of the realized bi-power. The first part of regressors (Morning, Lunch, Lunch 1, Afternoon, Aft. 1 and Evening) corresponds to the intra-daily parts of the realized bi-power. The second part of regressors (MorningJ, LunchJ, Lunch 1 J, AfternoonJ, Aft. 1 J and EveningJ) corresponds to the intra-daily parts of the realized jump.

Table 8.1

Statistics for the Residuals

	Morning RBP			Lunch RBP			Lunch RBP 1			Afternoon RBP			Afternoon RBP 1			Evening RBP		
Lag	AC	Q-stat.	p-value	AC	Q-stat.	p-value	AC	Q-stat.	p-value	AC	Q-stat.	p-value	AC	Q-stat.	p-value	AC	Q-stat.	p-value
1	0.023	0.6637	0.415	0.015	0.2639	0.607	0.015	0.2639	0.607	-0.018	0.4075	0.523	-0.002	0.0049	0.944	0.001	0.0016	0.968
2	-0.019	1.1021	0.576	-0.024	0.9744	0.614	-0.024	0.9744	0.614	-0.031	1.5791	0.454	-0.015	0.2625	0.877	-0.023	0.632	0.729
3	-0.021	1.6588	0.646	-0.015	1.2395	0.744	-0.015	1.2395	0.744	0.002	1.5864	0.662	-0.026	1.0518	0.789	-0.018	1.038	0.792
4	-0.037	3.3409	0.502	-0.019	1.6614	0.798	-0.019	1.6614	0.798	-0.012	1.7627	0.779	-0.019	1.4884	0.829	0.031	2.1921	0.700
5	-0.031	4.4809	0.482	0.015	1.9451	0.857	0.015	1.9451	0.857	0.038	3.5402	0.617	-0.006	1.5379	0.909	-0.032	3.4536	0.630
6	-0.024	5.1587	0.524	0.005	1.9747	0.922	0.005	1.9747	0.922	-0.025	4.3147	0.634	-0.018	1.9197	0.927	-0.014	3.7042	0.717
7	0.045	7.6692	0.363	-0.018	2.3557	0.938	-0.018	2.3557	0.938	0.035	5.7688	0.567	0.042	4.0275	0.777	0.064	8.6194	0.281
8	0.008	7.7497	0.458	0.026	3.1971	0.921	0.026	3.1971	0.921	0.03	6.8477	0.553	0.011	4.1835	0.840	-0.006	8.6608	0.372
9	0.028	8.6927	0.466	0.034	4.5978	0.868	0.034	4.5978	0.868	0.025	7.5974	0.575	-0.018	4.5922	0.868	0.039	10.56	0.307
10	-0.015	8.9559	0.536	0.005	4.6251	0.915	0.005	4.6251	0.915	0.014	7.8211	0.646	-0.033	5.9177	0.822	0.008	10.648	0.386

a) the upper table presents the restricted and restricted MEM models for the intra-daily volatility components. Restricted models are obtained by performing a Wald test and discarding regressors which are jointly insignificant at 5% level. The standard errors are reported in brackets and the coefficeints which are significant at 10%, 5 % and 1% level are marked with *,** and ***, respectively.

b) the bottom table presents the autocorrelation coefficients, Q-statistics and the associated p-values for the residuals of the corresponding MEM models up to lag 10.

Table 9

MEM Estimation for the RJ (Realized Jump) with Lagged Values of the Realized Bi-Power

	Morning RJ		Lunch RJ		Lunch 1 RJ		Afternoon RJ		Afternoon 1 RJ		Evening RJ	
Regressors	Full model	Restricted model	Full model	Restricted model	Full model	Restricted model	Full model	Restricted model	Full model	Restricted model	Full model	Restricted model
Intercept	1.92E-07 (4.2E-08)	1.95E-07 (3.9E-08)	4.19E-07 (1.2E-07)***	4.44E-07 (9.7E-08)***	2.03E-07 (4.7E-08)***	2.65E-07 (1.1E-07)***	4.08E-07 (1.3E-07)***	3.88E-07 (1.7E-07)**	1.85E-07 (2.61E-07)	2.22E-09 (1.5E-07)	4.03E-08 (1.2E-07)	1.01E-08 (6.6E-08)
CE	0.6880 (0.0369)***	0.7043 (0.0314)***	0.3108 (0.1041)***	0.3040 (0.1106)***	0.8157 (0.0387)***	0.7822 (0.0395)***	0.6309 (0.0694)***	0.6549 (0.0601)***	0.6246 (0.0988)***	0.6695 (0.0816)***	0.7426 (0.0959)***	0.7346 (0.0942)***
Morning	-0.0024 (0.0291)	0.1842 (0.0294)***	0.0401 (0.0440)	-----------	-0.0285 (0.0318)	-----------	0.0408 (0.0787)	-----------	0.3352 (0.1920)*	0.4134 (0.1285)***	-0.0777 (0.0929)	-----------
Lunch	-0.0292 (0.0233)	0.0652 (0.0293)**	-0.0044 (0.0297)	-----------	0.0583 (0.0397)	-----------	0.2188 (0.1378)	0.2178 (0.0671)***	0.0670 (0.1570)	-----------	-0.0469 (0.0762)	-----------
Lunch 1	0.0028 (0.0176)	-----------	0.0184 (0.0169)	-----------	-0.0169 (0.0164)	-----------	-0.0019 (0.0443)	-----------	-0.0010 (0.0743)	-----------	-0.0003 (0.0341)	-----------
Afternoon	0.0094 (0.0090)	-----------	0.0419 (0.0157)***	0.0289 (0.0118)**	0.0197 (0.0197)	-----------	-0.0278 (0.0291)	-----------	-0.0082 (0.0542)	-----------	0.0009 (0.0225)	-----------
Aft. 1	0.0342 (0.0124)***	0.0398 (0.0093)***	-0.0103 (0.0100)	-----------	0.0125 (0.0102)	-----------	0.0553 (0.0437)	0.0666 (0.0198)***	0.0218 (0.0758)	0.0978 (0.0444)**	0.0174 (0.0348)	-----------
Evening	0.0191 (0.0147)	-----------	-0.0012 (0.0168)	-----------	-0.0104 (0.0010)	-----------	0.0575 (0.0436)	-----------	-0.0277 (0.1021)	-----------	0.0468 (0.0710)	-----------
MorningB	0.0975 (0.0244)***	0.1138 (0.0209)***	0.0616 (0.0353)*	0.0951 (0.0281)***	-0.0087 (0.0204)	-----------	-0.0896 (0.0534)*	-----------	0.0186 (0.1299)	-----------	0.1743 (0.0956)*	0.1537 (0.0716)**
LunchB	0.0633 (0.0303)**	0.0545 (0.0240)**	0.0470 (0.0337)	-----------	0.0305 (0.0291)	-----------	-0.1300 (0.0495)***	-0.1450 (0.0496)***	-0.0389 (0.1952)	-----------	0.0722 (0.0988)	-----------
Lunch 1 B	0.0079 (0.0179)	-----------	-0.0150 (0.0146)	-----------	-0.0046 (0.0160)	-----------	0.1007 (0.0772)	0.1195 (0.0312)***	-0.0310 (0.1104)	-----------	0.0145 (0.0552)	-----------
AfternoonB	-0.0020 (0.0109)	-----------	-0.0183 (0.0107)*	-----------	0.0257 (0.0149)*	0.0487 (0.0123)***	0.1052 (0.0436)**	0.0926 (0.0359)***	-0.0086 (0.0582)	-----------	-0.003 0 (0.0296)	-----------
Aft. B 1	-0.0034 (0.0074)	-----------	0.0139 (0.0094)	-----------	-0.0079 (0.0063)	-----------	0.0135 (0.0272)	-----------	0.0222 (0.0540)	-----------	-0.0099 (0.0244)	-----------
EveningB	0.0131 (0.0111)	-----------	0.0341 (0.0137)**	0.0373 (0.0127)***	0.0215 (0.0103)*	0.0163 (0.0059)***	0.0170 (0.0395)	-----------	0.1351 (0.0873)	-----------	0.0094 (0.0495)	-----------

a) the upper table presents the restricted and restricted MEM models for the intra-daily volatility components. Restricted models are obtained by performing a Wald test and discarding regressors which are jointly insignificant at 5% level. The standard errors are reported in brackets and the coefficeints which are significant at 10%, 5 % and 1% level are marked with *,** and ***, respectively. With CE I denote the lagged value of the conditional expectation of the realized jump. The first part of regressors (Morning, Lunch, Lunch 1, Afternoon, Aft. 1 and Evening) corresponds to the intra-daily parts of the realized jump. The second part of regressors (MorningB, LunchB, Lunch 1 B, AfternoonB, Aft. 1 B and EveningB) corresponds to the intra-daily parts of the realized bi-power.

Table 9.1

Statistics for the Residuals

	Morning RBP			Lunch RBP			Lunch RBP 1			Afternoon RBP			Afternoon RBP 1			Evening RBP		
Lag	AC	Q-stat.	p-value	AC	Q-stat.	p-value	AC	Q-stat.	p-value	AC	Q-stat.	p-value	AC	Q-stat.	p-value	AC	Q-stat.	p-value
1	0.003	0.0097	0.922	0.015	0.283	0.595	0.015	0.2742	0.601	-0.018	0.3985	0.528	-0.001	0.0009	0.976	-0.002	0.0054	0.941
2	0.004	0.025	0.988	-0.011	0.4403	0.802	0.002	0.2778	0.87	0.009	0.4992	0.779	-0.004	0.0222	0.989	-0.006	0.0484	0.976
3	-0.017	0.3783	0.945	-0.021	0.9729	0.808	-0.059	4.4924	0.213	-0.018	0.8976	0.826	-0.013	0.2127	0.976	-0.016	0.3728	0.946
4	-0.015	0.6638	0.956	0.022	1.5413	0.819	-0.022	5.0753	0.28	0.001	0.8996	0.925	0.012	0.3795	0.984	-0.021	0.931	0.92
5	-0.014	0.9006	0.97	-0.027	2.4142	0.789	0.008	5.1587	0.397	0.042	3.0846	0.687	-0.008	0.4629	0.993	-0.024	1.6175	0.899
6	0.013	1.1195	0.981	0.019	2.8361	0.829	0.015	5.4153	0.492	0.022	3.6627	0.722	-0.004	0.481	0.998	-0.002	1.6206	0.951
7	-0.03	2.2113	0.947	0.002	2.8417	0.899	-0.027	6.2909	0.506	-0.006	3.7105	0.812	0.059	4.7224	0.694	0.044	3.9306	0.788
8	0.008	2.2876	0.971	-0.017	3.1957	0.921	-0.008	6.3781	0.605	0.06	8.1441	0.42	0.007	4.7807	0.781	0.022	4.5017	0.809
9	0.052	5.5347	0.785	-0.015	3.4574	0.943	0.017	6.722	0.666	-0.012	8.3193	0.502	0.023	5.4506	0.793	-0.031	5.6816	0.771
10	-0.039	7.4032	0.687	-0.013	3.6747	0.961	0.036	8.3075	0.599	-0.014	8.5487	0.575	-0.01	5.5729	0.85	0.01	5.804	0.831

c) the upper table presents the restricted and restricted MEM models for the intra-daily volatility components. Restricted models are obtained by performing a Wald test and discarding regressors which are jointly insignificant at 5% level. The standard errors are reported in brackets and the coefficeints which are significant at 10%, 5 % and 1% level are marked with *,** and ***, respectively.

d) the bottom table presents the autocorrelation coefficients, Q-statistics and the associated p-values for the residuals of the corresponding MEM models up to lag 10.

Table 10

Forecasting Performance of the Component Realized Volatility, Realized Range and Realized Power MEM Models

Forecasting Horizon in Days

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Realized Volatility	0.96	0.91	0.88	0.83574	0.79	0.77	0.74	0.72	0.70	0.68	0.66	0.64	0.63	0.61	0.60	0.58	0.57	0.56	0.54	0.53
Realized Range	0.97	0.96	0.97	0.96	0.94	0.92	0.90	0.87	0.84	0.82	0.80	0.79	0.77	0.75	0.73	0.72	0.71	0.69	0.67	0.66
Realized Power	0.97	0.94	0.92	0.88	0.84	0.82	0.80	0.78	0.76	0.74	0.73	0.71	0.70	0.68	0.67	0.66	0.64	0.63	0.61	0.60

a) the numbers in the table give the ratio of the average RMSE for the MEM models for the daily volatility measures versus the average RMSE of the corresponding component MEM models.

Table 11

Forecasting Performance of the Realized Bi-Power (RBP) and Realized Jump (RJ) Models

	Forecasting Horizon in Days																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Model 1[1]	0.95	0.92	0.87	0.82	0.77	0.73	0.71	0.69	0.66	0.65	0.63	0.61	0.60	0.58	0.57	0.56	0.55	0.54	0.52	0.51
Model 2[2]	0.98	0.96	0.89	0.83	0.78	0.74	0.71	0.69	0.66	0.64	0.62	0.60	0.59	0.57	0.56	0.55	0.54	0.53	0.52	0.51
Model 3[3]	0.95	0.92	0.88	0.83	0.79	0.77	0.75	0.73	0.72	0.71	0.70	0.69	0.69	0.68	0.68	0.67	0.66	0.66	0.65	0.65
Model 4[4]	0.96	0.92	0.90	0.87	0.85	0.82	0.80	0.78	0.76	0.74	0.72	0.71	0.69	0.68	0.66	0.65	0.63	0.62	0.60	0.59
Model 5[5]	0.95	0.92	0.90	0.86	0.82	0.78	0.75	0.73	0.70	0.68	0.66	0.64	0.62	0.60	0.58	0.57	0.56	0.54	0.53	0.52
Model 6[6]	0.94	0.90	0.87	0.82	0.78	0.74	0.72	0.70	0.69	0.67	0.66	0.64	0.63	0.62	0.60	0.59	0.58	0.57	0.56	0.55

[1] This row of the table gives the ratio of the RMSE of the daily MEM model for the RBP versus the RMSE for the component MEM model for the RBP
[2] This row of the table gives the ratio of the RMSE of the daily MEM model for the RBP versus the RMSE for the daily MEM model for the RBP with lagged values of the daily RJ
[3] This row of the table gives the ratio of the RMSE of the daily MEM model for the RBP versus the RMSE for the component MEM model for the RBP with lagged values of the intra-daily components of the RJ
[4] This row of the table gives the ratio of the RMSE of the daily MEM model for the RJ versus the RMSE for the component MEM model for the RJ
[5] This row of the table gives the ratio of the RMSE of the daily MEM model for the RJ versus the RMSE for the daily MEM model for the RJ with lagged values of the daily RBP
[6] This row of the table gives the ratio of the RMSE of the daily MEM model for the RJ versus the RMSE for the component MEM model for the RJ with lagged values of the intra-daily components of the RBP

Table 12

Forecasting Performance of the Combined Realized Bi-Power (RBP) and Realized Jump (RJ) MEM Models

	Forecasting Horizon in Days																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Model 1[1]	1.00	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
Model 2[2]	0.95	0.91	0.88	0.84	0.80	0.77	0.75	0.73	0.71	0.69	0.67	0.65	0.64	0.62	0.61	0.60	0.58	0.57	0.56	0.55
Model 3[3]	0.97	0.95	0.90	0.85	0.80	0.76	0.73	0.70	0.67	0.65	0.63	0.61	0.59	0.58	0.57	0.55	0.54	0.54	0.52	0.51
Model 4[4]	0.94	0.90	0.87	0.82	0.78	0.75	0.73	0.72	0.70	0.69	0.68	0.67	0.66	0.65	0.64	0.63	0.63	0.62	0.62	0.61

[1] This row of the table gives the ratio of the RMSE of the daily MEM model for the Realized Volatility (RV) versus the RMSE for the MEM model for the RV using the combined MEM models for the daily RBP and RJ without the interaction effects.
[2] This row of the table gives the ratio of the RMSE of the daily MEM model for the Realized Volatility (RV) versus the RMSE for the MEM model for the RV using the combined MEM models for the daily RBP and RJ with the interaction effects.
[3] This row of the table gives the ratio of the RMSE of the daily MEM model for the Realized Volatility (RV) versus the RMSE for the MEM model for the RV using the combined MEM models for the component RBP and RJ without the interaction effects.
[4] This row of the table gives the ratio of the RMSE of the daily MEM model for the Realized Volatility (RV) versus the RMSE for the MEM model for the RV using the combined MEM models for the component RBP and RJ with the interaction effects.